Particle Physics

Prof. Sharam Rahatlou

Author: Angelo Serrecchia

A.Y. 2023/2024

Introduction

INTRODUCTION

CONVENTIONS / RELATIONS

- $\underline{P}$: 4-vector $\quad \underline{P} = (E, \vec{p}) \quad E^2 = p^2 + m^2 \quad |\vec{p}| = p$

- metric's matrix $\quad g^{\mu\nu} = \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}$

- <u>NATURAL UNITS:</u> $\hbar = c = 1 \longrightarrow [T] = [L] = [E]^{-1}$

  $\hbar c = 200$ MeV.fm

- $1\,eV = 1{,}6 \cdot 10^{-19}\,J$ $\quad q = e \quad \Delta V = 1V \quad \Delta K = q\Delta V = 1\,eV$

- $\beta = \dfrac{|\vec{p}|}{E}$; $\gamma = \dfrac{E}{m} \longrightarrow \beta\gamma = \dfrac{|\vec{p}|}{m}$

$\boxed{\text{DECAY}}$: QM process where $a \longrightarrow b + c + \ldots + z$

- ~~$a \to b$~~ violates energy conservation
- $a \to b + c$ (2 bodies decay)
- $a \to b + c + d$ (3 bodies decay)
- $a \to b + c + \ldots + z$ ($n \geq 2$ bodies decay)

$E^2 = p^2 + m^2 \longrightarrow$ allows the presence of creation and annihilation operators

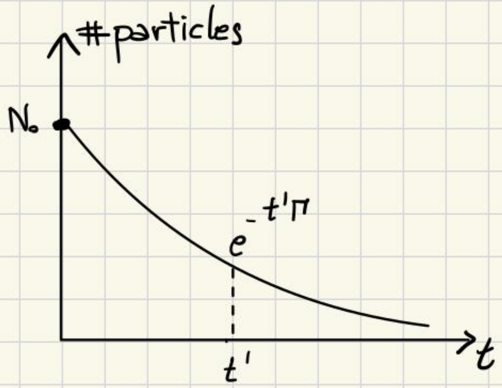
Very well know decays:

- $n \longrightarrow p\, e^- \overline{\nu}_e$
- $X^A_Z \longrightarrow Y^{A-4}_{Z-2} + \alpha \qquad \alpha \equiv He^4_2$
- $K^0 \longrightarrow \pi^+ \pi^-$
- $\mu^- \longrightarrow e^- \overline{\nu}_e \nu_\mu$
- $\pi^0 \longrightarrow \gamma\gamma$

WHAT ARE THE LAWS WHICH GOVERNS A DECAY ?

Let's take an initial sample of N_0 particles. After a certain time t the # of particles is

★ $\boxed{N(t) = N_0 e^{-\Gamma t}}$ # particles at t



$\longrightarrow \boxed{\dfrac{N(t)}{N_0} < 1 \quad : \text{survey probability}}$

What's the meaning of Γ?

- $[\Gamma] = T^{-1} = E$ (eV) (in natural units)
- Γ is the probability of decay per unit time. It is called decay width.

Mean Life Time

We could also write ★ as:

$\boxed{N(t) = N_0 e^{-\frac{t}{\tau}}} \longrightarrow \boxed{\tau \equiv \Gamma^{-1}}$

"average" (not the exact life time)

$\begin{cases} \tau : \text{mean life time (sec)} \\ \Gamma : \text{decay width (MeV)} \end{cases}$

Since τ is a mean, it can be computed as:

$$\boxed{\tau = \langle t \rangle = \frac{\int_0^\infty dt\, e^{-\Gamma t}\, t}{\int_0^\infty e^{-\Gamma t} dt} = \frac{1}{\Gamma}}$$

Energetic possible decay

In order to know if a decay is energetic possible we have to know the Q-VALUE.

Let's put in the reference frame where $\vec{P}_a = 0$ (rest frame of particle a)

$\longrightarrow$ Since : $E_a = E_b + E_c \longrightarrow m_a = m_b + K_b + m_c + K_c$

In the limit of $K_b, K_c \longrightarrow 0 \implies m_a \geq m_b + m_c$

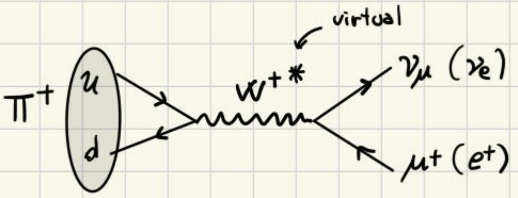
Therefore we can define :

$\boxed{Q =: m_a - m_b - m_c \geq 0}$ for decay to happen

In general for a N-body decay:

$$\boxed{Q = m_a - \sum_{i=1}^{N} m_i}$$

<u>Example</u>: $\pi^+ \longrightarrow \mu^+ \nu_\mu$ or $e^+\nu_e$



$\pi^+ \longrightarrow \mu^+ + \nu_\mu \quad Q = m_\pi - m_\mu \simeq 34$ MeV

$\pi^+ \longrightarrow e^+ + \nu_e \quad Q = m_\pi - m_e \simeq 139{,}5$ MeV

<u>Example</u>: $\pi^0 \longrightarrow \mu^+\mu^-$? $\quad Q < 0 \rightarrow$ not possible

How to estimate Γ (or τ)?

We know that Γ can be computed with the Fermi-Golden rule

$$\Gamma(i \rightarrow f) = 2\pi \, |\mathcal{M}|^2 \rho(E) \Big|_{E_f = E_i}$$

<u>Fermi's 2^{nd} Golden rule</u>

$|\mathcal{M}|^2$: depends on Interaction Hamiltonian $\mathcal{M} = \langle f | H_I | i \rangle$

$\rho(E)$: Density of states (The Q value is related to the volume of the phase state)

Relativistic Golden Rule

Let's consider: $\quad 1 \longrightarrow 2 + 3 + \ldots + n$

$$\Gamma = S \cdot \frac{1}{2m_1} \int |\mathcal{M}_{fi}|^2 \, \delta^{(4)}(\underline{p}_1 - \underline{p}_2 - \underline{p}_3 - \ldots - \underline{p}_n) \cdot \prod_{j=2}^{n} (2\pi) \, \delta(p_j^2 - m_j^2) \, \theta(E_j) \frac{d^4 p_j}{(2\pi)^4}$$

S: statistical factor

$\delta(p_j^2 - m_j^2)$: j-th particle is on-shell

<u>What is the S-factor?</u> It is a purely <u>numerical factor</u>:

Let's consider $\quad a \longrightarrow b + b + c + c + c \longrightarrow S = \frac{1}{N_b!}\frac{1}{N_c!} = \frac{1}{2!}\frac{1}{3!} = \frac{1}{12}$

$\quad a \longrightarrow b + c \longrightarrow S = 1$

Example: 2 body decay $\quad a \rightarrow b + c \quad (S = 1)$

$$\Gamma = \frac{1}{2m_a} \int |\mathcal{M}|^2 (2\pi)^4 \delta^4(\underline{p}_a - \underline{p}_b - \underline{p}_c) \cdot \prod_{j=b,c} (2\pi) \, \delta(p_j^2 - m_j^2) \, \theta(E_j) \frac{d^4 p_j}{(2\pi)^4}$$

<u>How to compute?</u> $\delta(p_j^2 - m_j^2)\, \theta(E_j)$

- $f(E) \equiv p_j^2 - m_j^2 = E_j^2 - |p_j|^2 - m_j^2 = E_j^2 - (\vec{p}_j^{\,2} + m_j^2) \longrightarrow f'(E) = 2E \quad E_{1,2} = \pm\sqrt{p^2 + m^2}$
- I can use that $\delta(f(x)) = \sum_i \frac{\delta(x - x_i)}{|f'(x)|_{x = x_i}}$

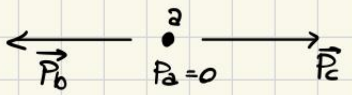
$$\longrightarrow \delta(p_j^2 - m_j^2)\,\theta(E_j) = \delta\left(E_j - \sqrt{p_j^2 + m_j^2}\right) = \frac{\delta(E-E_j)}{2E_j} + \underbrace{\cancel{\frac{\delta(E+E_j)}{2E_j}}}_{\theta(E_j)} = \frac{\delta(E-E_j)}{2\sqrt{p_j^2+m_j^2}}$$

How to compute $\frac{d^4p_j}{(2\pi)^4}$ = ? $\longrightarrow \frac{d^4p_j}{(2\pi)^4} = \frac{dE_j\, d^3p_j}{(2\pi)^4}$

$$\longrightarrow \Gamma_{a\to b+c} = \frac{1}{8(2\pi)^2}\frac{1}{2m_a}\int |\mathcal{M}|^2\, \delta^4(\underline{p}_a - \underline{p}_b - \underline{p}_c)\, \frac{1}{4\sqrt{p_b^2+m_b^2}\sqrt{p_c^2+m_c^2}}\, d^3p_b\, d^3p_c$$

How to compute $\delta^4(\underline{p}_a - \underline{p}_b - \underline{p}_c)$? $\quad \delta^4(\underline{p}_a - \underline{p}_b - \underline{p}_c) = \delta(E_a - E_b - E_c)\,\delta^3(\vec{p}_a - \vec{p}_b - \vec{p}_c)$

In the rest frame of a



$$\longrightarrow p_a = 0 \longrightarrow \begin{cases} \delta(\vec{p}_b + \vec{p}_c) \longrightarrow \vec{p}_b = -\vec{p}_c \\ \delta(E_a - E_b - E_c) = \delta(m_a - E_b - E_c) \end{cases} \longrightarrow |\vec{p}_b| = |\vec{p}_c| = p$$

$$\longrightarrow \boxed{\Gamma_{a\to b+c} = \frac{1}{32\pi^2}\frac{1}{m_a}\int |\mathcal{M}|^2\, \frac{\delta(m_a - E_b - E_c)}{\sqrt{p_b^2+m_b^2}\sqrt{p_c^2+m_c^2}}\, d^3p}$$

If I define $\boxed{u = E_b + E_c} = \sqrt{p^2+m_b^2} + \sqrt{p^2+m_c^2}$

$$\longrightarrow \frac{du}{dp} = \frac{2p}{2\sqrt{\ }} + \frac{2p}{2\sqrt{\ }} = \frac{p\left(\sqrt{\ } + \sqrt{\ }\right)}{\sqrt{\ }\cdot\sqrt{\ }} = \frac{pu}{\sqrt{p^2+m_b^2}\sqrt{p^2+m_c^2}}$$

$$\longrightarrow \int |\mathcal{M}|^2\, \delta(m_a - u)\, \frac{1}{u}\, \frac{du}{dp}\, p\, dp = \int |\mathcal{M}|^2\, \delta(m_a - u)\, \frac{p}{u}\, du = |\mathcal{M}|^2\, \frac{1}{m_a}\, p$$

$$\longrightarrow \boxed{\Gamma = \frac{1}{8\pi}\frac{1}{m_a^2}|\mathcal{M}|^2\, |\vec{p}|}$$

phase space (relation with Q-value, see example)

Example (relation between phase space and Q-value)

$Q(\pi^+ \longrightarrow e^+\nu_e) = 34$ MeV
$Q(\pi^+ \longrightarrow \mu^+\nu_\mu) = 139{,}5$ MeV $\quad\longrightarrow |\vec{p}_e| \gg |\vec{p}_\mu|$

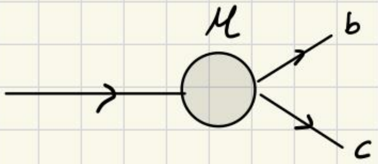
$$\longrightarrow \frac{\Gamma(\pi^+ \to \mu^+\nu_\mu)}{\Gamma(\pi^+ \to e^+\nu_e)} = \frac{|\vec{p}_\mu|}{|\vec{p}_e|}\frac{|\mathcal{M}_{\pi\mu}|^2}{|\mathcal{M}_{\pi e}|^2} \ll 1$$

Exercise $a \longrightarrow b + c + d$

Dimensional analysis :

Lecture 2 28/02/2024

$$\Gamma = \frac{1}{8\pi}\,\frac{1}{m_a}\,\frac{1}{m_b}\,|\mathcal{M}|^2\,|\vec{p}\,|$$



in a ref. frame

$P_b = P_c = P \qquad p = |\vec{p}\,|$

$$\mathcal{M}^2 = |\langle f | H_I | i \rangle|^2$$

Is it correct? It is correct from dimensional analysis $[\Gamma] = E$ ✓

Typical particle involved : $e^-, p, n, \nu_e, \dots$

FUNDAMENTAL PARTICLES of the S.M.

$$\begin{array}{cccc}
 & 1^{st} & 2^{nd} & 3^{rd} \\
\begin{array}{c} -1 \\ 0 \end{array} & \begin{pmatrix} e^- \\ \nu_e \end{pmatrix} & \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} & \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix} \\
\begin{array}{c} +\frac{2}{3} \\ -\frac{1}{3} \end{array} & \begin{pmatrix} u \\ d \end{pmatrix} & \begin{pmatrix} c \\ s \end{pmatrix} & \begin{pmatrix} t \\ b \end{pmatrix}
\end{array}
\begin{array}{l} \text{leptons} \\ \\ \text{quarks} \end{array}$$

$\xrightarrow{\text{mass}}$

For the leptons : E.M. , weak

For the quarks: E,M. , Weak, Strong

In nature there are Hadrons: composite particles of quarks
- Baryons: $q_1 q_2 q_3$
- Mesons : $q_1 \bar{q}_2$

Antiparticle: state which has all the opposite quantum number respect to the associated particle.

Example : e^+ : anti-particle of e^-
$\bar{u}$: anti-particle of u

$$\begin{array}{cccc}
 & 1^{st} & 2^{nd} & 3^{rd} \\
\begin{array}{c} 0 \\ +1 \end{array} & \begin{pmatrix} \bar{\nu}_e \\ e^+ \end{pmatrix} & \begin{pmatrix} \bar{\nu}_\mu \\ \mu^+ \end{pmatrix} & \begin{pmatrix} \bar{\nu}_\tau \\ \tau^+ \end{pmatrix} \\
\begin{array}{c} +1/3 \\ -2/3 \end{array} & \begin{pmatrix} \bar{d} \\ \bar{u} \end{pmatrix} & \begin{pmatrix} \bar{s} \\ \bar{c} \end{pmatrix} & \begin{pmatrix} \bar{b} \\ \bar{t} \end{pmatrix}
\end{array}
\begin{array}{l} \text{anti leptons} \\ \\ \text{antiquarks} \end{array}$$

Examples

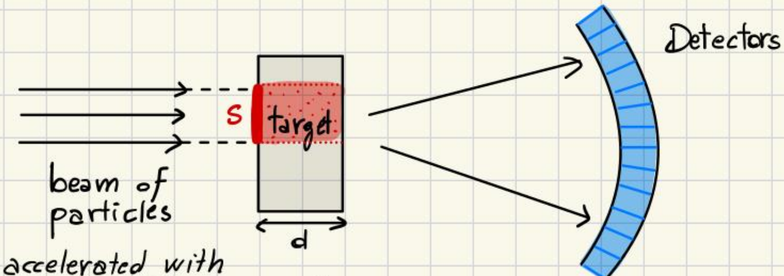
$$\left.\begin{array}{l}
\mu^- \longrightarrow e^- \bar{\nu}_e \nu_\mu \\
\pi^- \longrightarrow \mu^- \bar{\nu}_\mu \\
n \longrightarrow p\, e^- \bar{\nu}_e
\end{array}\right\} \text{unstable particles}$$

$\pi^- : \bar{u} d \qquad Q = -\frac{2}{3} + \left(-\frac{1}{3}\right) = -1$

α - particles: $\alpha = {}^4_2He$ $\quad Q = +2$

COLLISIONS $\quad a + b \longrightarrow c + d + e + \dots + z$

▶ FIXED TARGET



beam of particles
accelerated with an $\underline{E}$ field, deflected with an $\underline{H}$ field

How do we count the reactions? $\quad \dfrac{\#\ \text{reactions}}{\Delta T}$

$$\frac{dN_r}{dt} = \frac{\#\ \text{reactions}}{\text{time of the exp.}} = \sigma \frac{dN_p}{dt} n_b \cdot d$$

σ → cross section (Has to do with nature of interaction)

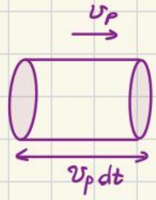
n_b → density of the targets $\quad [n_b] = L^{-3}$

$[\sigma] = L^2 \quad (b) \qquad 1\text{ barn} = 10^{-24}\text{cm}^2$

$$\frac{dN_r}{dt} = \sigma \cdot \frac{dN_p}{dt} \underbrace{n_b \cdot d \cdot \frac{S}{S}}_{} = \sigma\, N_B \cdot \frac{dN_p}{dt \cdot S}$$

$n_b \cdot d \cdot S$ = # Targets

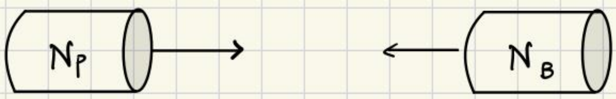
$\dfrac{dN_p}{dt \cdot S}$ ← flux of incoming particles Φ_p



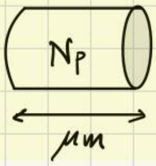
$\phi_p = n_p \cdot v_p$

$$\boxed{\frac{1}{N_B}\frac{dN_r}{dt}} = \sigma \cdot \Phi_p \quad \propto \mathcal{P}(\text{interaction}) \qquad \Gamma(i \rightarrow f) \quad : \frac{\text{prob}}{\text{unit time}}$$

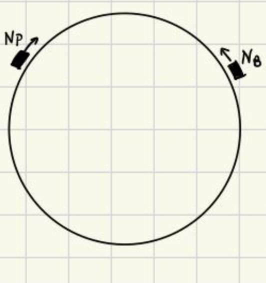
▶ MOVING TARGET (COLLIDING BEAMS) $\quad$ (greater $\sqrt{s}$ → more convenient)



How do we produce p? Ionizing Hidrogen with an $\vec{E}$ field (The binding energy is 13,6 eV)
At LHC in a packet we have 10^{11} protons



To do the collision I have to accelerate two packets in opposite dir. : we do this with 2 opposite $\vec{B}$.



$$\frac{dN_r}{dt} = \sigma N_B \Phi_p = \sigma N_B \frac{N_p}{S} f_{rev} = \sigma \frac{N_B \cdot N_p}{S} f_{rev}$$

f_{rev} → frequence of revolution

We have to take into account the effective section

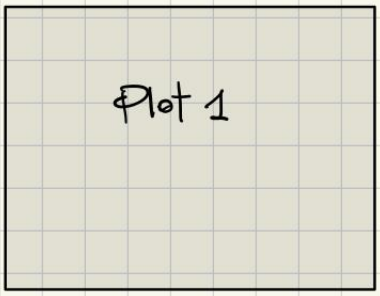
Instantaneous luminosity $\boxed{\mathcal{L} = \frac{N_B \cdot N_P}{S} f_{rev}}$

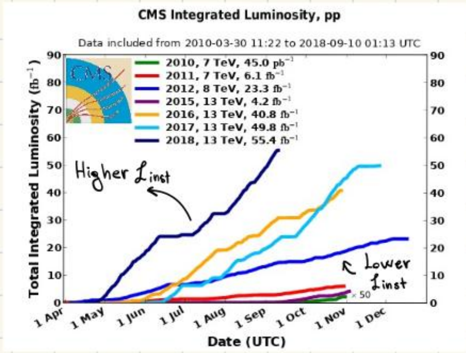
$$\frac{dN_r}{dt} = \sigma \cdot \mathcal{L} \qquad [\mathcal{L}] = cm^{-2} s^{-1} = b^{-1} s^{-1}$$

Integrated luminosity $\boxed{L = \mathcal{L}_{inst} \cdot \Delta T}$

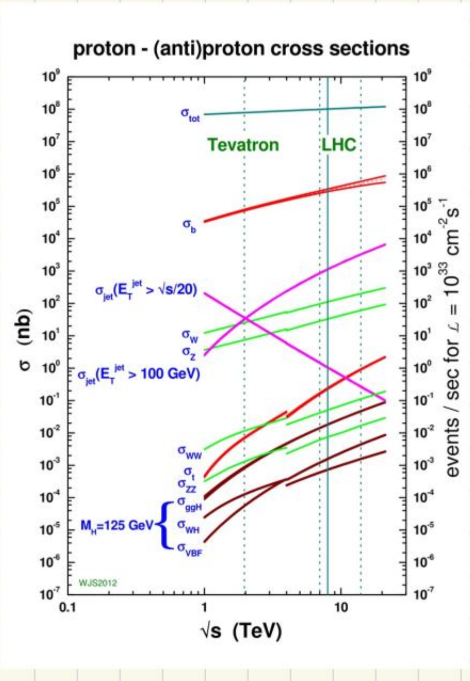
$$N_{events} = \frac{dN_r}{dt} \Delta t = \sigma \cdot \mathcal{L}_{inst} \cdot \Delta T = \sigma \cdot L \qquad [L] = b^{-1}$$

Example LHC





BEHAVIOUR OF σ



$$P_1 = (E_1, \vec{p}) \qquad P_2 = (E_2, -\vec{p})$$

$$\sqrt{s} = \sqrt{(P_1 + P_2)^2} = \sqrt{(2E)^2} = 2E$$

LHC now has $\boxed{E = 6{,}5 \text{ TeV}}$ Stunning!

$$\sigma_{tot} = 10^8 \, nb \qquad L = 1 \, nb^{-1}$$

$$\longrightarrow \# \text{ events} = 10^8$$

$\sigma(p + p \longrightarrow H + x) = 10^{-2} \, nb \longrightarrow$ In order to produce 1H we need $L = 100 \, nb^{-1}$!

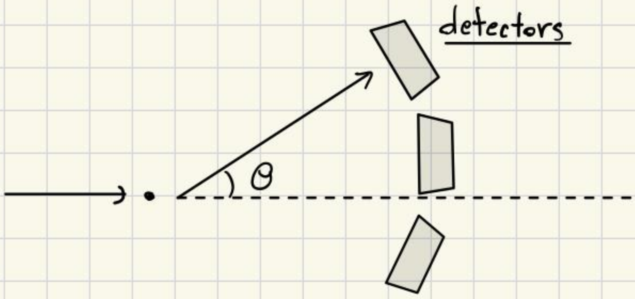
$\sigma(p+p \longrightarrow H+X)$ inclusive cross-section

$\sigma(p+p \longrightarrow p+p+p+\bar{p})$ exclusive cross-section

Differential cross section : # events as a function of something

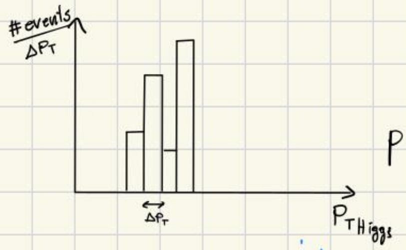
- <u>Angular dependence</u>:

  - $\dfrac{d\sigma}{d\theta}$ or $\dfrac{d\sigma}{d\varphi}$ or $\dfrac{d\sigma}{d\Omega}$



- <u>Energetic dependence</u>:

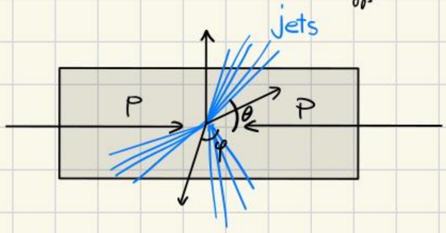
  - $\dfrac{d\sigma}{dP_{T(H)}} = \dfrac{\#\,\text{Higgs}}{\Delta P_{T(\text{Higgs})}}$



$p+p \longrightarrow H+X$

- <u>Angular and energetic dependence</u>:

  - $\dfrac{dN}{dP_T\, d\theta\, d\varphi}$



Channels of the Higgs' hunting

<u>Example</u> (zz channel)

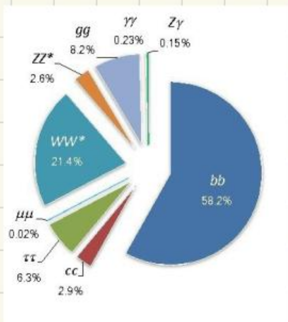
$$p+p \longrightarrow H+X$$
$$H \longrightarrow Z^\circ Z^\circ$$
$$Z^\circ \longrightarrow e^+e^-$$
$$Z^\circ \longrightarrow e^+e^-$$

$$N(p+p \longrightarrow H+X \rightarrow 4e+X) = \sigma_H \cdot L \cdot BF(H \rightarrow ZZ) \times BF(Z \rightarrow ee) \times BF(Z \rightarrow ee)$$

<u>BRANCHING FRACTIONS</u>

$H \longrightarrow$	$b\bar{b}$	Γ_1	channel 1
	$c\bar{c}$	Γ_2	channel 2
	ZZ	Γ_3	channel 3
	$\gamma\gamma$	Γ_4	channel 4
	$\vdots$	$\vdots$	$\vdots$

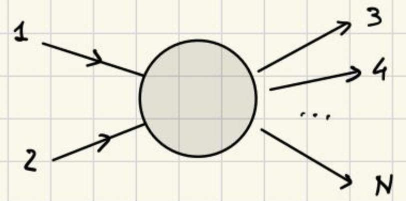
$$\boxed{BF_i = \frac{\Gamma_i}{\Gamma_{tot}}}$$



<u>Example</u> (μμ channel)

$$\longrightarrow N(p+p \longrightarrow H+X \longrightarrow (4\mu)+X) = \sigma \cdot L \cdot (3\cdot 10^{-2})\cdot(3\cdot 10^{-2})\cdot(3\cdot 10^{-2}) \simeq \sigma \cdot L \cdot 3\cdot 10^{5}$$

Fermi Golden rule for scattering $\quad 1+2 \longrightarrow 3+4+\dots+N$



$$\boxed{\sigma = \frac{S}{4\sqrt{(\underline{p}_1\cdot\underline{p}_2)^2-(m_1 m_2)^2}}\int |\mathcal{M}|^2 (2\pi)^4\,\delta^4(\underline{p}_1+\underline{p}_2-\underline{p}_3\dots-\underline{p}_N)\times\prod_{j=3}^{N}\frac{1}{2\sqrt{p_j^2+m_j^2}}\,\frac{d^3\vec{p}_j}{(2\pi)^3}}$$

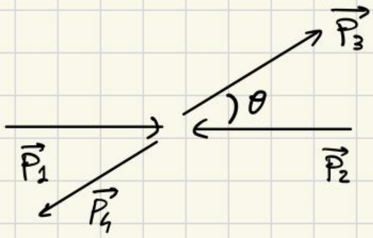
Example: $\quad 1+2 \longrightarrow 3+4$

Center of mass: reference frame where the total momentum is zero.

$$\text{CMS} \longleftrightarrow \vec{p}_1+\vec{p}_2=0$$

(N.B. in LHC since $|\vec{p}_1|=|\vec{p}_2|$ CMS $\equiv$ LAB : convenient for calculus)

The conservation of momentum tells us that $\vec{p}_1+\vec{p}_2=0=\vec{p}_3+\vec{p}_4$



$p_1 = p_2 = p_{in}$
$p_3 = p_4 = p_{fin}$

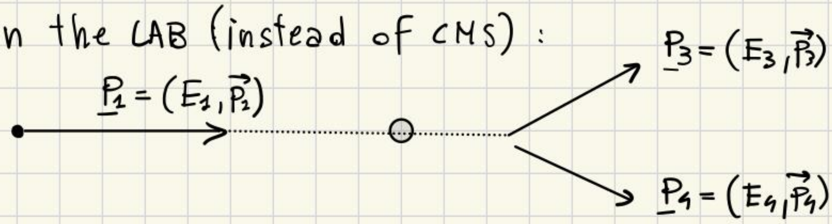
$$\sigma = \frac{1}{4\sqrt{(\vec{p}_1\cdot\vec{p}_2)^2-(m_1m_2)^2}}\int|\mathcal{M}|^2\,\delta^4(\underline{p}_1+\underline{p}_2-\underline{p}_3-\underline{p}_4)\times(2\pi)^4\cdot\frac{1}{(2\pi)^6}\,\frac{d^3\vec{p}_3}{\sqrt{p_3^2+m_3^2}}\,\frac{d^3\vec{p}_4}{\sqrt{p_4^2+m_4^2}}$$

- $\delta^4(\underline{p}_1+\underline{p}_2-\underline{p}_3-\underline{p}_4) = \delta(E_1+E_2-E_3-E_4)\,\delta^3(\underbrace{\vec{p}_1+\vec{p}_2}_{=0}-\vec{p}_3-\vec{p}_4)$
- $\dfrac{\delta(E_1+E_2-E_3-E_4)}{\sqrt{p_{out}^2+m_3^2}\sqrt{p_{out}^2+m_4^2}}\,d^3\vec{p}_{out}$
- $u = E_3+E_4$ (change of variable)
- $d^3\vec{p}_{out} = p_{out}^2\,dp_{out}\cdot d\Omega$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{S}{64\pi^2}\,\frac{1}{(E_1+E_2)^2}\,\frac{|\vec{p}_{out}|}{|\vec{p}_{in}|}\,|\mathcal{M}|^2}$$

(N.B. being a diff. cross. section it depends on the ref. frame $\rightarrow$ it is valid only in the CMS)

Example: if we are in the LAB (instead of CMS):



$$\longrightarrow \boxed{\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\frac{1}{m_2^2}\frac{|\vec{P}_{out}|}{|\vec{P}_{in}|}|\mathcal{M}|^2} \qquad \text{(Valid in the LAB)} \quad *$$

Examples of 2 body scattering

Rutherford : $\alpha + N_i \longrightarrow \alpha + N_f$ $\qquad \frac{m_\alpha}{m_N} \sim \frac{4}{79}$

Compton : $\gamma + e^- \longrightarrow \gamma + e^-$

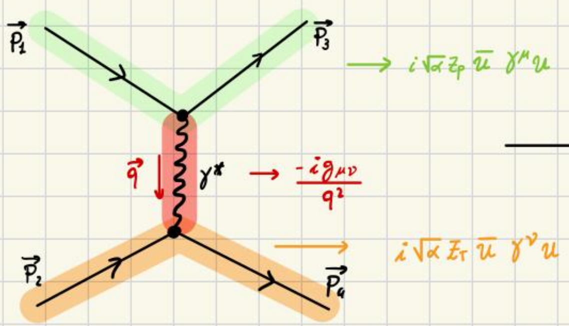
$e^- + p$: $e^- + p \longrightarrow e^- + p$ $\qquad \frac{m_e}{m_p} \sim 2000$

OBSERVATION:

In the Rutherford exp. : $K \simeq 5\,\text{MeV}$ $(M_\alpha \sim 4\,\text{GeV})$ $\rightarrow$ α is non relativistic

In an e^- experiment if $K \simeq 5\,\text{MeV}$ $(M_{e^-} \sim 0{,}5\,\text{MeV})$ $\longrightarrow$ α is relativistic (we have to take into account spinors)

MOTT FORMULA (cross section in the case of a 2 body scattering in the case of $m_T \gg m_P$)

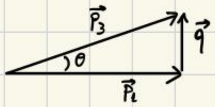


$$\longrightarrow \mathcal{M} \sim \frac{1}{q^2} \longrightarrow |\mathcal{M}|^2 \sim \frac{1}{q^4} \qquad \text{where } q = P_3 - P_1 = (E_3 - E_1, \vec{P}_3 - \vec{P}_1)$$

- if $\boxed{m_1 \ll m_2}$ $\longrightarrow \vec{P}_4 \simeq 0$ (Negligible recoil). Taking also the target at rest $\vec{P}_2 \simeq 0$:

$$\longrightarrow |\vec{P}_1| \equiv |\vec{P}_{in}| \simeq |\vec{P}_3| \equiv |\vec{P}_{out}|$$

There's only a deflection, not a change in magnitude:



$$\longrightarrow |\vec{q}| = 2|\vec{P}_1|\sin\frac{\theta}{2} \longrightarrow q^4 = 16\,|\vec{P}_{in}|^4 \sin^4\frac{\theta}{2}$$

Doing computations one can find the Mott's formula:

$$|\mathcal{M}|^2 = \left(\frac{\alpha\, m_2}{q^2}\right)^2 \left(m_1^2 + |\vec{P}_{in}|^2 \cos^2\frac{\theta}{2}\right)$$

Calling with $m_T \equiv m_2$ (mass of the target) and $m_P \equiv m_1$ (mass of projectile), from $*$ we find:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\frac{1}{\cancel{m_T^2}}\left(\frac{\alpha\,\cancel{m_T}}{q^2}\right)^2\left(m_P^2 + |\vec{P}_{in}|^2\cos^2\frac{\theta}{2}\right)\cancel{\frac{|\vec{P}_{out}|}{|\vec{P}_{in}|}} \simeq$$

$$\simeq \frac{1}{64\pi^2}\left(\frac{\alpha}{q^2}\right)^2\left(m_P^2 + P_{in}^2\cos^2\frac{\theta}{2}\right) =$$

$$= \frac{1}{64\pi^2}\left(\frac{\alpha}{P_{in}^2\sin^2\frac{\theta}{2}}\right)^2\left(m_P^2 + P_{in}^2\cos^2\frac{\theta}{2}\right)$$

$$\longrightarrow \boxed{\frac{d\sigma}{d\Omega} \simeq \frac{1}{64\pi^2}\left(\frac{\alpha}{p_{in}^2 \sin^2\frac{\theta}{2}}\right)^2 \left(m_p^2 + p_{in}^2 \cos^2\frac{\theta}{2}\right)}$$

Mott formula : $m_T \gg m_p$

We can easily retrieve the Rutherford cross section considering the non relativistic limit : $m_1 \gg |\vec{p}_{in}|$:

$$\longrightarrow \frac{d\sigma}{d\Omega} \simeq \frac{1}{64\pi^2}\left(\frac{\alpha}{p_{in}^2 \sin^2\frac{\theta}{2}}\right)^2 m_p^2$$

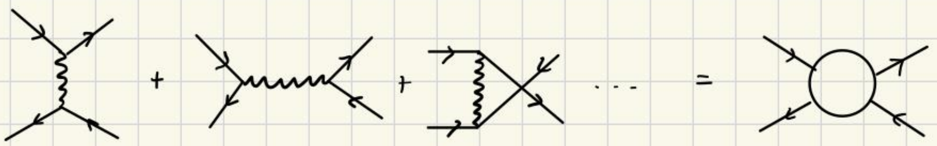
Rewriting of Mott's formula :

$$\begin{aligned} m_1^2 + p_{in}^2 \cos^2\frac{\theta}{2} &= m_1^2 + p_1^2 \left(1 - \sin^2\frac{\theta}{2}\right) = \\ &= m_1^2 + p_1^2 - p_1^2 \sin^2\frac{\theta}{2} = \\ &= E_1^2 - p_1^2 \sin^2\frac{\theta}{2} = \\ &= E_1^2 \left(1 - \left(\frac{p_1}{E_1}\right)^2 \sin^2\frac{\theta}{2}\right) = E_1^2 \left(1 - \beta^2 \sin^2\frac{\theta}{2}\right) \end{aligned}$$

$$\longrightarrow \frac{d\sigma}{d\Omega} \simeq \frac{1}{64\pi^2}\left(\frac{\alpha}{q^2}\right)^2 E_1^2 \left(1 - \beta^2 \sin^2\frac{\theta}{2}\right) \equiv \left(\frac{d\sigma}{d\Omega}\right)_{Ruth} \cdot \left(1 - \beta^2 \sin^2\frac{\theta}{2}\right)$$

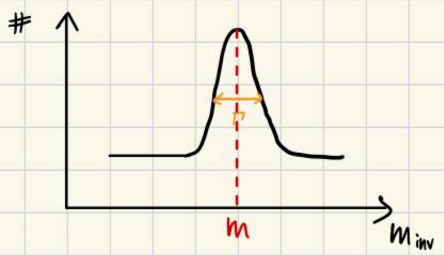
Thanks to this formula we know the emission angle of the particles in the final state so exploiting the angle dependance of $\frac{d\sigma}{d\Omega}$ we know how to build the detector.

Another important thing to consider is that during a collision different things could happen



we have a black box in the middle

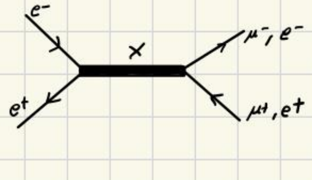
In the middle we could also have a new particle but not a stable one. Unstable particles are characterized by their mass and their lifetime, or equivalently by the decay width Γ (and the spin J). So denoting with $X(m, \Gamma, J)$ an unstable particle we can count the number of decays and what we see is a Breit-Wigner distribution:



the formula of the cross section is:

$$\sigma(i \rightarrow x) = \frac{2J+1}{(2S_1+1)(2S_2+1)} \frac{4\pi}{|\vec{p}_{in}|^2} \frac{\Gamma_{tot}^2}{4(E-E_0)^2 + \frac{\Gamma}{4}} \frac{\Gamma_{in}}{\Gamma_{tot}} \frac{\Gamma_{out}}{\Gamma_{tot}}$$

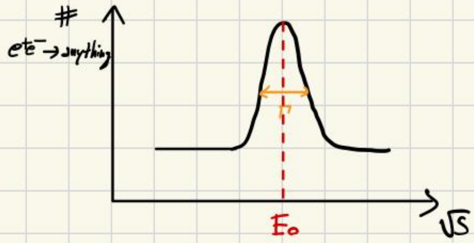
Let's look at the case of $e^+e^- \rightarrow \mu^+\mu^-$ (e^+e^-)



$\longleftrightarrow$ $m_x \geq 2m_e, 2m_\mu$

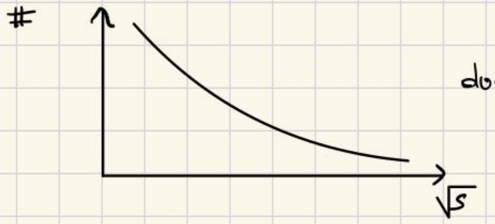
Let's say that we have in the final state $\mu^+\mu^-$ and we ask ourself whether we can see e^+e^- in the final state or not. Since we se $\mu^+\mu^-$ the diagram works well, so the coupling X with electrons is not $\varnothing$ $\longrightarrow$ so it's possible to see e^+e^- in the final state.

So, if we have a resonance in our graph we see:

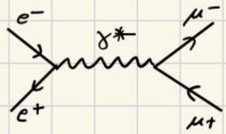


$$\sqrt{s} = \sqrt{(P_1 + P_2)^2} = \sqrt{(E_1+E_2)^2 - (\vec{P}_1+\vec{P}_2)^2} = E_1 + E_2$$

If instead we do not have a resonance we see:



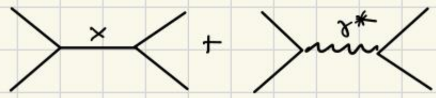
due to



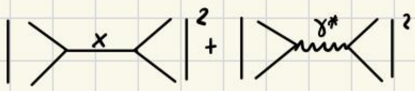
n.b. $\mathcal{M} \sim \frac{1}{q^2} = \frac{1}{s}$

So in nature we have:

$e^+e^- \longrightarrow \mu^+\mu^-$ $\quad \mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2 =$

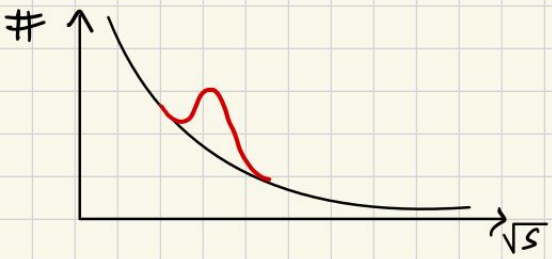


$\longrightarrow \sigma =$



$+ \mathcal{M}_1\mathcal{M}_2^* + \mathcal{M}_1^*\mathcal{M}_2$

So we see the production of a resonance as a Breit Wigner peak on the top of what we expect without a resonance:



- We observe $N = N_S + N_B$
- We estimate $N_S = N - N_B^{est}$ with $N_B^{est} = N_B \pm \sqrt{N_B}$
- If $\dfrac{N^{obs} - N_B^{est}}{\sqrt{N_B^{est}}}$ $> 5\sigma$: observation! 3σ: evidence!

Probing structure of nucleons

Deviation in alpha scattering
Electron-nucleon scattering, elastic and inelastic limit
Mandelstar and Bjorken variables
Form factor
Rosenbluth formula
Experimental proof of proton structure
Deep inelastic scattering
Probing structure functions at SLAC
Quark parton model
Parton density functions

Symmetries

C-parity
P-parity
Isospin: hypothesis and experimental evidence
G-parity
Strangeness
Gellman-Nishijima formula

Static quark model

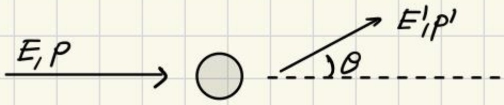
Mesons multiplets
Production and decays of rho0
Baryon multiplets
Prediction and discovery of Omega-
Hypothesis of new quantum number: color
Baryon and. Meson wave functions
Gluons
Extension to more quarks

Using QED to prove the quark model

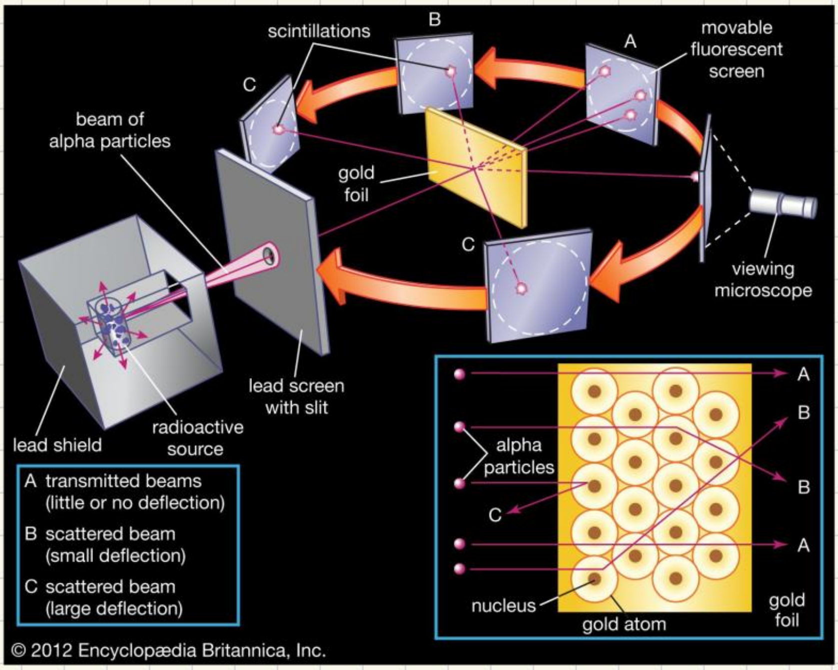
Bhabha and Möller scattering
Muon pair production
Quark Pair (hadron) production
Ratio R of hadronic and muon cross section
Experimental tests: PETRA, JADE
Discovery of J/Psi and properties
Decay of qqbar resonances: Phi(1020), J/Psi
Discovery of the b quark and Upsilon resonances
Discovery of the t quark
Discovery of the tau lepton

RUTHERFORD SCATTERING (1910)

Lecture 4 : 04/03/2024



→ E' and θ are the observable quantities



$$\frac{d\sigma}{d\Omega} \propto \frac{\#\,\text{events}}{\Delta\theta} \quad \text{v.s. } \theta$$

$$d\Omega = \sin\theta \, d\theta \, d\varphi = 2\pi \sin\theta \, d\theta$$

($d\varphi$ does not play any role, because of the central potential)

Rutherford result $\longrightarrow$ $\boxed{R_N < 10 \text{ fm}}$

Compatible with $R_N = r_0 A^{1/3}$
($r_0 \sim 1$ fm , nuclear radius)

From a theoretical point of view we know that in general :

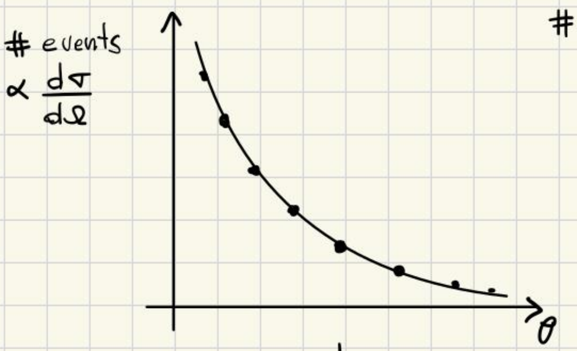
$$\frac{d\sigma}{d\Omega} = \underbrace{(\ldots)(Z_p Z_T \alpha)^2 \frac{1}{q^4} \frac{|\vec{P}_{out}|}{|\vec{P}_{in}|} E'^2}_{\text{Rutherford piece}} \underbrace{\left(1 - \beta^2 \sin^2\frac{\theta}{2}\right)}_{\text{Mott piece}}$$

Mott's formula

However we observe that

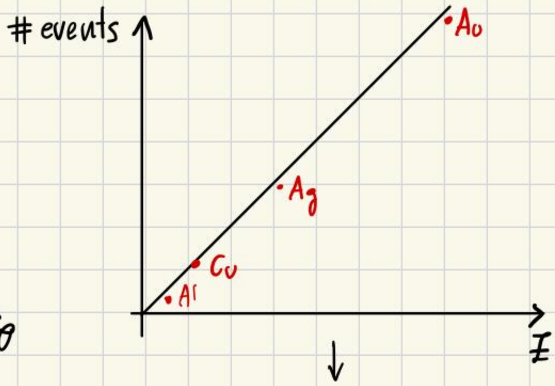
1) $K_\alpha \simeq 5$ MeV , $M_\alpha \simeq 3,7$ GeV $\longrightarrow$ $\boxed{\frac{P_{out}}{P_{in}} \simeq 1 \quad \text{negligible recoil}}$

2) $\beta \ll 1 \longrightarrow \boxed{\frac{d\sigma}{d\Omega} \simeq \left(\frac{d\sigma}{d\Omega}\right)_{Ruth} \sim \frac{1}{(P_{in})^4 \sin^4\frac{\theta}{2}}}$



↓

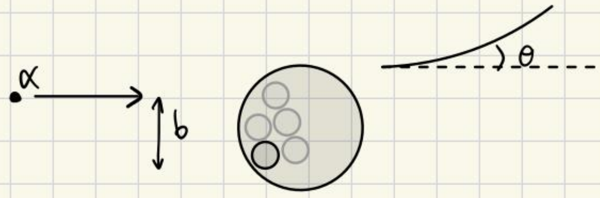
Q.E.D. Hint is working



↓

\# events is scaling with the Z_{target}

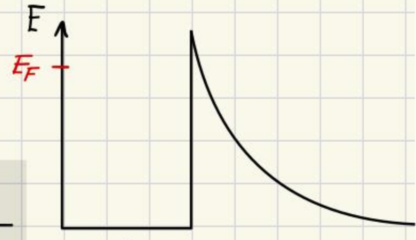
Why Rutherford cannot be used to infer the structure of the nucleus?



5 MeV are not enough to infer the properties of the nucleus.

I need to increase the energy. Let's see the process of α creation (α-decay):

$$\boxed{{}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + \alpha}$$



Nuclear potential

Fermi Gas model for nucleus ⟶ Fermi energy $E_F = \dfrac{P_F^2}{2M_N}$

In order to see the structure I need to have:

$$P_F \simeq 200 \text{ MeV} \longrightarrow E_F \simeq 20 \text{ MeV}$$

Okay, but how we know $P_F \simeq 200$ MeV? We want to see a $\Delta x \sim 1$ fm

$\Delta p \cdot \Delta x \sim 1 \longrightarrow \hbar c \simeq 200 \text{ MeV fm} = 1 \longrightarrow 200 \text{ MeV} \simeq 1 \text{ fm}^{-1}$

In order to see the nucleus structure we need probes of 20-30 MeV (not easy to do)

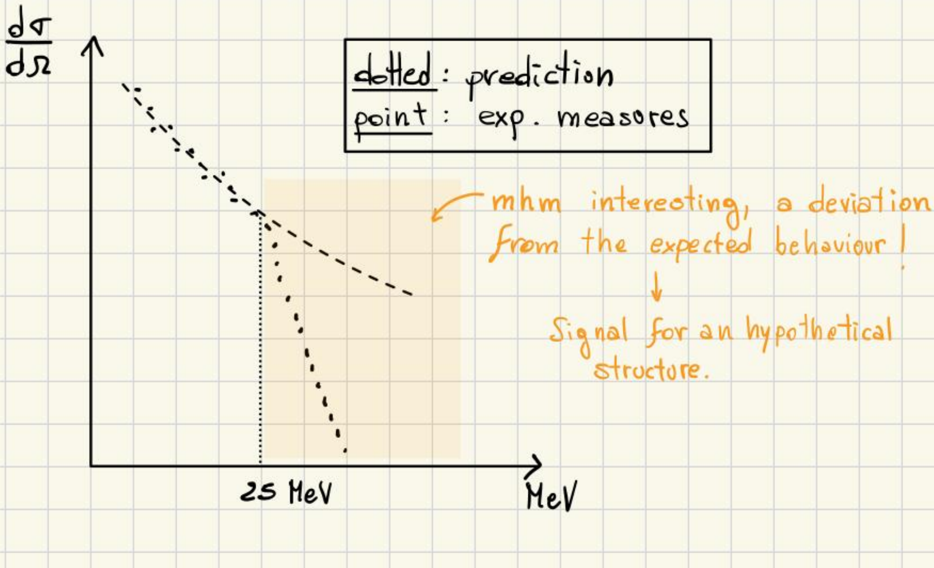
The best thing to do however is use pointlike particles! e.g.: $e^- + p$

Okay, but at least can we say with Rutherford if is there a structure or not inside the nucleus?

$\dfrac{d\sigma}{d\Omega}$ should go $\propto \dfrac{\alpha^2 (Z_P Z_T)^2}{P_{in}^4 \sin^4\frac{\theta}{2}} E'^2 \left(1-\beta^2 \sin^2\frac{\theta}{2}\right) \longrightarrow$ it assumes that $H_I = $ E.M. and pointlike probe on pointlike target

If $\left.\dfrac{d\sigma}{d\Omega}\right|_{Meas} \neq \left.\dfrac{d\sigma}{d\Omega}\right|_{expected} \longrightarrow$ H_I is wrong or the target is not pointlike

Rev. Mod. Phys 33, 190 (1961)



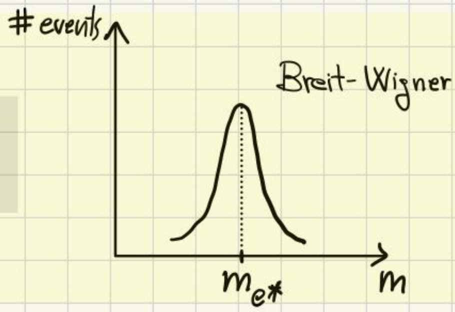
How can I say that something is elementar?

A composite particles has more-internal d.o.f. i.e. angular momentum, energy excitations. Therefore if I transfer energy to that particle, if it comes out that it acquires all the energy in kinetic form it means that it is pointlike.

How to observe the presence of excitation, example:

$$p + p \longrightarrow e^* + X$$
$$e^* \longrightarrow e^- + \gamma$$

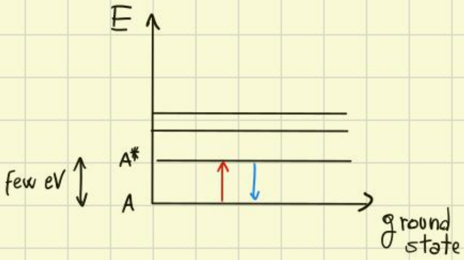
$$m_{inv} = \sqrt{(\underline{P}_\gamma + \underline{P}_{e^-})^2}$$



e^- + Target ($E_e \gtrsim 25$ MeV)

How can I say that the target is a proton or an atom A?

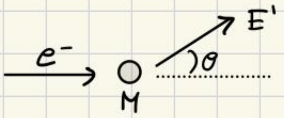
Suppose $e^- + A \longrightarrow \ldots$



$A^* \longrightarrow A + \gamma$

→ If I send an e^- with 25 MeV there's no way to have an atomic interaction (of course statistically speaking)

High energy electron beam ~1960



Possible processes:

$e^- + p/N \longrightarrow e^- + p/N$	elastic scattering
$e^- + p/N \longrightarrow e^- + p^*/N^*$	excitation and then emission
$e^- + N \longrightarrow e^- + X$	breaking of target

OBSERVABLES: E', θ of deflected electron

$$\left.\frac{d\sigma}{d\Omega}\right|_{Mott} \sim \alpha^2 \frac{1}{q^4} E'^2 \left(1 - \beta^2 \sin^2\frac{\theta}{2}\right)$$

We have to take into account also the spin of the electron (Rutherford does not take it into account). Mott's formula instead takes it into account.

Ultra-relativistic limit : $E_e \gg m_e \rightarrow \beta \sim 1$

$$\longrightarrow \frac{d\sigma}{d\Omega} \propto \frac{\alpha^2}{q^4} E'^2 \underbrace{\left(1-\sin^2\frac{\theta}{2}\right)}_{\cos^2\frac{\theta}{2}} = \frac{\alpha^2}{q^4} E'^2 \cos^2\frac{\theta}{2}$$

- $\theta = 0$ $\quad Max\left(\frac{d\sigma}{d\Omega}\right) \longrightarrow$ it goes to fast that basically it does not interact $\xrightarrow{e^-} \circ \xrightarrow{e^-}$
- $\theta = \pi$ $\quad \frac{d\sigma}{d\Omega} \sim \cos^2\left(\frac{\pi}{2}\right) = 0$ $\quad \xrightarrow{e^-} \circ$, $\xleftarrow{e^-}$

Why for $\theta = 0$ $\frac{d\sigma}{d\Omega} \sim 0$?

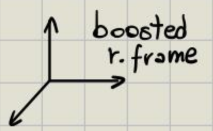
Suppose :

initial state: $\xrightarrow{e^-} \bullet$ $\qquad$ final state: $\xleftarrow{e^-} \bullet$

Helicity : $\boxed{h = \frac{\vec{p}\cdot\vec{s}}{|\vec{p}|\cdot|\vec{s}|}}$ $\quad$ for massive particles it is not Lorentz invariant

$h = -1$ $\quad \xrightarrow{e^-} \bullet$ $\quad$ is it possible to swap h changing my reference frame?

Yes! The key is to use a boost transformation i.e. go faster than the e^-
$\rightarrow \beta' > \beta_e$



final state: $h = +1$ $\;\leftarrow$

$$\beta' > \beta_{e^-} = \frac{p}{E}$$

This does not work for a massless particle : h is a Lorentz invariant . i.e. its value is the same in every ref. frame. Helicity is an intrinsic property for the massless particle and it is equivalent to chirality.

Chirality : $\boxed{\gamma_5}$:

$$\psi(r) = \underbrace{\frac{1}{2}(1+\gamma^5)}_{P_L}\psi + \underbrace{\frac{1}{2}(1-\gamma^5)}_{P_R}\psi$$

For ultra relativistic particles : helicity $\simeq$ chirality

Example :

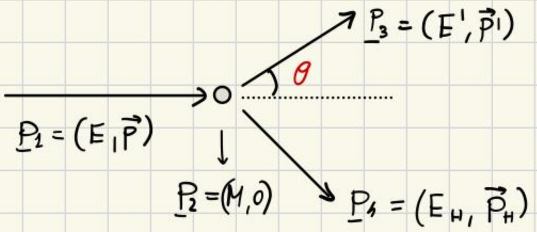
e^-: $\beta \sim 1$ $\qquad$ initial: $\xrightarrow{e^-} \bullet$ $\qquad$ final: $\xleftarrow{e^-} \bullet$ $\quad \rightarrow$ I changed the spin! Impossible

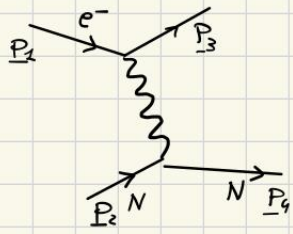
Therefore $\theta = \pi$ is not possible in $e^- + N \rightarrow e^- + N$ with ultra-relativistic electrons

N.B. $\quad \frac{d\sigma}{d\Omega} \sim \frac{\alpha^2}{q^4} E'^2 \left(1-\beta^2 \sin^2\frac{\theta}{2}\right)$ includes spin (but not recoil for target)

spin ½ relativistic probe on target (no recoil)

Elastic limit: $\underline{P}_4 = (M, \vec{P}_4 \simeq 0)$ (no recoil) $e^- + N \longrightarrow e^- + N$





Observables: E, E', θ

$$\underline{P}_1 + \underline{P}_2 = \underline{P}_3 + \underline{P}_4 \longrightarrow \underline{P}_4 = \underbrace{\underline{P}_1 - \underline{P}_3}_{q} + \underline{P}_2$$

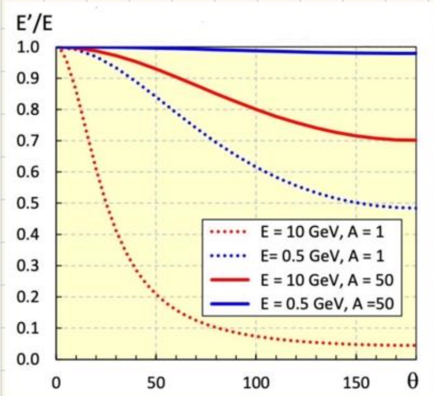
$E_1 - E_3, \ \vec{P}_1 - \vec{P}_3$

$(E_1 - E_3)^2 - |\vec{P}_1 - \vec{P}_3|^2$

$2m_e^2 - 2E_1E_3 + 2P_1P_2\cos\theta$

$$|\underline{P}_4|^2 = M^2 \longrightarrow (\underline{P}_1 - \underline{P}_3 + \underline{P}_2)^2 = m_e^2 + M^2 + m_e^2 - 2\,\underline{P}_1 \cdot \underline{P}_3 + 2\,\underline{P}_1 \cdot \underline{P}_2 - 2\,\underline{P}_3 \cdot \underline{P}_2 \overset{!}{=} M^2$$

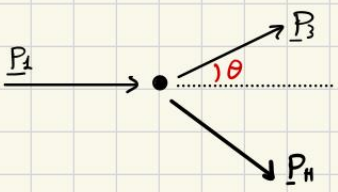
$$\longrightarrow \boxed{E' = \frac{E}{1 + \frac{E}{M}(1 - \cos\theta)} = \frac{E}{1 + 2\frac{E}{M}\sin^2\frac{\theta}{2}}}$$

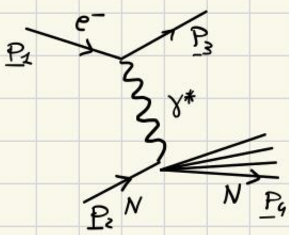


(Equivalent to Compton scattering)

$E_e \simeq 25$ MeV $\gg m_e \rightarrow m_e \simeq 0$

Inelastic limit $e^- + N \longrightarrow e^- + X$ (X: more particles than 1)





$\underline{P}_4 \equiv \underline{P}_H = \sum_i \underline{P}_{H_i}$ (particles in the final state)

$$\boxed{q^2 = 2m_e^2 - 2(EE' - EE'\cos\theta) \simeq -4EE'\sin^2\frac{\theta}{2}}$$

$E, E' > 0 \longrightarrow q^2 < 0$ by definition $\longrightarrow$ 4-moment of the virtual γ is space-like

MANDELSTAR VARIABLES

We can define the Mandelstan variables $\boxed{Q^2 =: -q^2 \equiv t}$ $\boxed{t = -(\underline{P}_1 - \underline{P}_3)^2}$; $\boxed{s = (\underline{P}_1 + \underline{P}_2)^2}$

(Q^2 tells us how much energy can we transfer to the final state)

Observations:

$$\begin{cases} t = -q^2 = -(\underline{P}_1 - \underline{P}_3)^2 \\ s = (\underline{P}_1 + \underline{P}_2)^2 = m_e^2 + M^2 + 2\underline{P}_1 \cdot \underline{P}_2 = m_e^2 + M^2 + 2ME = M^2 + 2ME \end{cases}$$

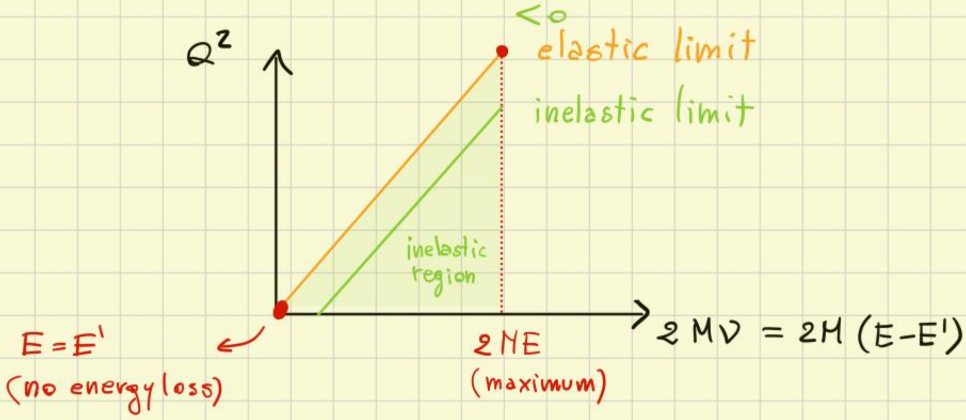
$P_1 = P_2 + q \xrightarrow{\text{if}} |P_1|^2 = |P_H|^2 = W^2$ invariant mass of all hadronic particles

$$\longrightarrow (P_2+q)^2 = M^2 - Q^2 + 2M\underbrace{(E-E')}_{\nu} \longrightarrow \boxed{W^2 = M^2 - Q^2 + 2M\nu}$$

inelastic general case

OBSERVATIONS ABOUT $Q^2 = M^2 - W^2 + 2M\nu$

- Elastic limit $\longrightarrow \boxed{W^2 = M^2} \longrightarrow Q^2 = 2M\nu$
- Inelastic : $\boxed{W^2 > M^2} \longrightarrow Q^2 = \underbrace{M^2 - W^2}_{<0} + 2M\nu$



BJORKEN VARIABLES

Experimentally we measure $E, E', \theta \longrightarrow$ all the variable are function of those.
We could define 2 new variables:

$$\boxed{x = \frac{Q^2}{2M\nu}} = \frac{M^2 - W^2 + 2M\nu}{2M\nu} = 1 + \frac{M^2 - W^2}{2M\nu} \qquad \begin{matrix} W^2 \geq M^2 \\ \nu \geq 0 \end{matrix} \rightarrow \boxed{0 \leq x \leq 1}$$

Bjorken variable

$$\boxed{Y = \frac{\nu}{E}} = \frac{E-E'}{E} \qquad 0 \leq E' \leq E \rightarrow \boxed{0 \leq Y \leq 1}$$

Examples: (Why they're useful)

- Elastic case $Q^2 = 2M\nu \longrightarrow \boxed{x = 1}$
- Inelastic case $\longrightarrow \boxed{x < 1}$

Physical meaning :
- x : energy transfer to targets
- Y : energy loss

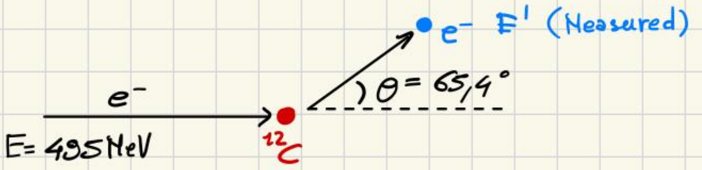
EXPERIMENT #1 :

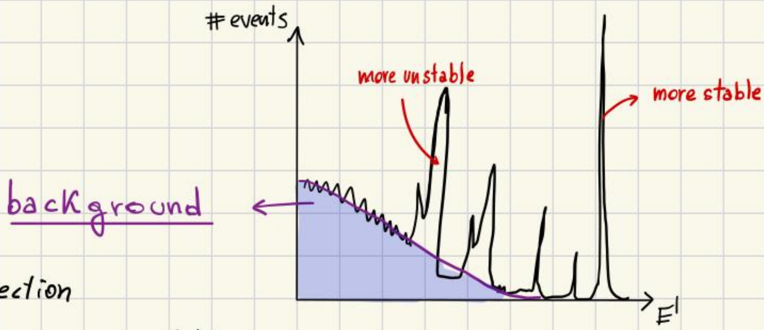
Electron beam : $E_e = p_e = 495$ MeV

Carbon target : $e^- + {}^{12}C \longrightarrow e^- + {}^{12}C$ $\qquad E' \ll E \qquad E'$: energy of scattered e^-

$e^- + {}^{12}C \longrightarrow e^- + {}^{12}C^*$ excitation

$e^- + {}^{12}C \longrightarrow e^- + X$





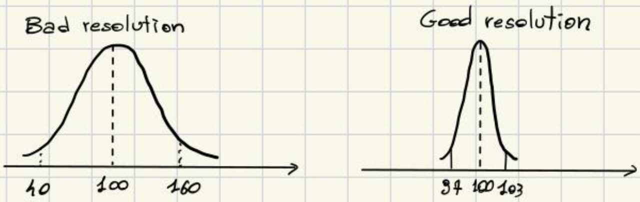
- <u>Different high</u>: different differential cross section
- <u>Different width</u>: $\boxed{\Gamma = \frac{1}{\tau}}$
  - Γ large τ small $\rightarrow$ more unstable
  - Γ small τ large $\rightarrow$ more stable

<u>Observed width</u>: $BW(E') \otimes$ Resolution $(E-E')$ (Good detector $\rightarrow$ narrow Gaussian resolution)

<u>Examples</u>:

- $BW \rightarrow \delta(E-M) \longrightarrow \int dx' \, \delta(x-x') \, e^{-\frac{(x-x'-\mu)}{\sigma^2}} = e^{\frac{-(x-\mu)}{\sigma^2}}$

- 2 Detectors with different resolution:



<u>Elastic (final peak)</u>

$$E' = \frac{E}{1 + 2\frac{E}{M}\left(\sin^2\frac{\theta}{2}\right)}$$

$E = 495$ MeV
$\theta = 65{,}4°$
$M \cong 12 M_p \simeq 12$ GeV

$$\longrightarrow E' = \frac{1}{1+0{,}03} E \cong (1-0{,}03) \cdot E \simeq 485 \text{ MeV} \quad \text{(consistent)}$$

In 1960 at SLAC: Stanford Linear Accelerator Center:

- $e^- + p \longrightarrow e^- + p$ $\quad p = uud$
- $e^- + p \longrightarrow e^- + \Delta^+(1332)$ excited barion (proton) uud
- $e^- + p \longrightarrow e^- + \pi^+ + n$
- $e^- + p \longrightarrow e^- + p + \pi^+ + \pi^-$
- $e^- + p \longrightarrow e^- + \Delta(1332) + \pi^0$
- $e^- + p \longrightarrow e^- + \Delta(1332) + \pi^+ + \pi^-$

UPDATE OF THE MOTT'S FORMULA

So far $\left.\frac{d\sigma}{d\Omega}\right|_{Mott} \sim (\dots)\,\frac{\alpha^2}{q^4} E'^2 \left(1-\beta^2 \sin^2\frac{\theta}{2}\right)$ It assumes point-like target, no target's spin and no recoil

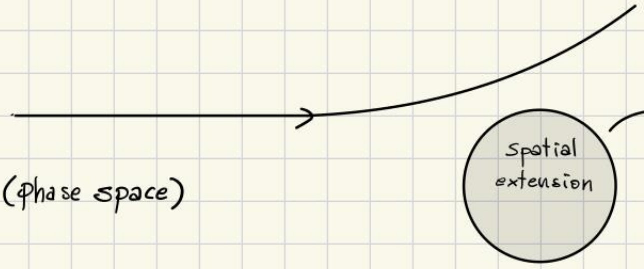
It's time to take all into account:

1) SPIN OF THE TARGET p, n $s=\frac{1}{2}$ particle
   From an E.M. point of view having $s \neq 0$ we have take into account the magnetic moment

2) STRUCTURE OF TARGET : no point-like

3) RECOIL OF TARGET

STRUCTURE FACTOR



How can we take into account the spatial extension?
The Form Factor is the Key!

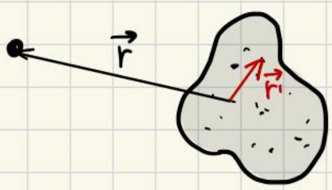
Remember: $\frac{d\sigma}{d\Omega} \propto |\mathcal{M}|^2 \times (\text{phase space})$

- $\mathcal{M} = \langle f | H_I | i \rangle$. Point like: $H_I = Z_p Z_T \frac{e^2}{r}$

- Born approximation: planar wave for initial and final free particles

$$\psi_e = \frac{1}{\sqrt{V}} e^{i\vec{p}\cdot\vec{r}} \longrightarrow \mathcal{M} \simeq \int d^3r\, e^{-i\vec{p}\cdot\vec{r}} e^{+i\vec{p}'\cdot\vec{r}} \frac{1}{r} = \int d^3r \frac{e^{i\vec{q}\cdot\vec{r}}}{r} \propto \frac{1}{q^2}$$

For an extended body:



$$V(\vec{r}) = \int d^3r' \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}$$

$\rho(\vec{r})$ charge distribution of target

- point-like $\rho(\vec{r}) = Z_T e\, \delta(\vec{r})$
- extended body $\rho(\vec{r}') = Z_T e f(\vec{r}')$

$$\longrightarrow \mathcal{M} \sim Z_p Z_T e^2 \int d^3r\, e^{i\vec{q}\cdot\vec{r}} \int d^3r' \frac{f(\vec{r}')}{|\vec{r}-\vec{r}'|}$$

$$\longrightarrow \mathcal{M} \sim \int d^3r \frac{e^{i\vec{q}\cdot(\vec{r}-\vec{r}')}}{|\vec{r}-\vec{r}'|} \underbrace{\int d^3r'\, e^{i\vec{q}\cdot\vec{r}'} f(\vec{r}')}_{\text{Form Factor}}$$

$$\boxed{F(q^2) := \int d^3r'\, e^{i\vec{q}\cdot\vec{r}'} f(\vec{r}')}$$

The Form Factor is the Fourier transform of spatial charge distribution:

$$\mathcal{M} \sim \frac{Z_p Z_T e^2}{q^2} F(q^2) \longrightarrow \frac{d\sigma}{d\Omega} \propto |\mathcal{M}|^2 \sim \left(\frac{Z_p Z_T e^2}{q^2}\right)^2 |F(q^2)|^2$$

$$\longrightarrow \boxed{\left.\frac{d\sigma}{d\Omega}\right|_{\substack{e^- \text{ on extended}\\ \text{target}}} = \left.\frac{d\sigma}{d\Omega}\right|_{\text{point-like}} \times |F(q^2)|^2}$$

Physical meaning of $F(q^2)$: more energy we put in the virtual particle more we're going to resolve the Q.M. aspects of the target. Higher is the energy the more resolution we have from a spatially point of view.

How to measure $F(q^2)$

Remember: $q^2 = -4EE' \sin^2\frac{\theta}{2}$.

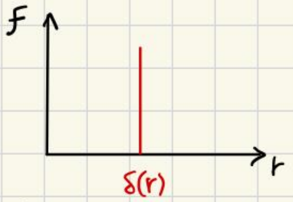
It is not practical to scan all q^2, in fact most accessible values are $q^2 \to 0$ because I could do an expansion of $F(q^2)$ around ϕ: (If I measure a lot of data near ϕ).

$$F(q^2) = \int d^3r \, e^{i\vec{q}\cdot\vec{r}} f(\vec{r}) \qquad \text{typically} \quad f(\vec{r}) \overset{\text{Central}}{=} f(r) = \int d\varphi \sin\theta \, d\theta \, dr \, r^2 \, e^{i\vec{q}\cdot\vec{r}} f(\vec{r})$$

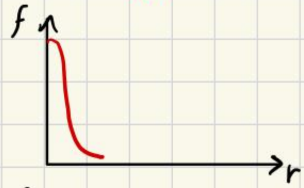
$$\longrightarrow \quad F(q^2) = \frac{4\pi}{A}\int_0^\infty dr \, r^2 \frac{\sin(q\cdot r)}{q\cdot r} f(r) \qquad A = \int_0^\infty d^3r \, f(r) = 4\pi \int_0^\infty dr \, r^2 f(r)$$

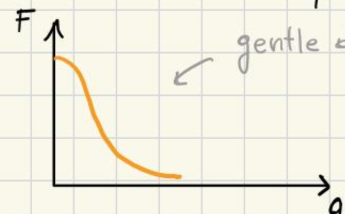
Examples of $f(r)$

1) point-like : $f(r) = e \frac{1}{4\pi}\delta(r)$



2) Gaussian: $f(r) = a^2 e^{-a^2\frac{r^2}{2}}$





3) Homogeneous: $f(r) = \frac{1}{\frac{4}{3}\pi R^3}$ $r \le R$

In conclusion we can infer the structure by looking the form of $F(q^2)$

Measure # events at q^2 and $q^2 \simeq 0 \longrightarrow$ Expand $F(q^2)$ at $q^2 = 0$

$$F(q^2) = (\text{Norm}) \int_0^\infty \int_{-1}^{1} \int_0^{2\pi} \underbrace{e^{iqr\cos\theta}}_{\substack{\simeq \\ 1 + iqr\cos\theta - \frac{1}{2}(qr)^2\cos^2\theta + o(qr)^3}} f(r) \, r^2 dr \, d\cos\theta \, d\varphi \simeq 1 - \frac{1}{6} q^2 \langle r^2 \rangle \quad \text{where } \langle r^2 \rangle = \int_0^\infty f(r) r^2 dr$$

We can define:

$$\boxed{R =: \frac{\left.\frac{d\sigma}{d\Omega}\right|_{\text{meas}}}{\left.\frac{d\sigma}{d\Omega}\right|_{\text{point like}}} \simeq |F(q^2)|^2 \simeq 1 - \frac{1}{6} q^2 \langle r^2 \rangle}$$

It is a very useful observable because:

if $R < 1 \longrightarrow \langle r^2 \rangle \neq 0$ (not point like)

SPIN OF TARGET

Dirac particle target (p, n, e)

Examples:

$$e^- + e^- \longrightarrow e^- + e^- \quad \text{Möller}$$

$$e^- + e^+ \longrightarrow e^- + e^+ \quad \text{Bhabha}$$

Spin $\xrightarrow{\text{implies}}$ magnetic moment $\boxed{\mu = g \dfrac{e\hbar}{4m}}$ $\qquad \mu_N = \dfrac{e\hbar}{4m_p}$: nuclear magneton

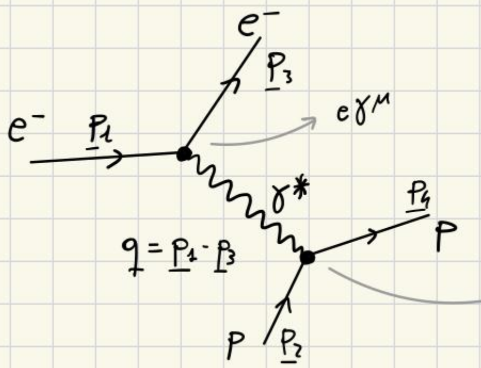
particle	mass	expected	measured
e^- :	$m = m_e$	$g = 2$	$g_e \simeq 2$
p :	$m = m_p$	$g = 2$	$g_p = 2{,}79$
n :	$m = m_n$	$g = 0$	$g_n = -1{,}91$

anomalous magnetic moment

$\downarrow$

indication of possible structure, not elementary

ROSENBLUTH FORMULA



$$\left[F_1(q^2)\, e\, \gamma^\nu + i \frac{\sigma^{\nu\tau} q_\tau}{2M} \kappa F_2(q^2)\right]$$

structure spatial charge distribution

magnetic moment, spin of target

$$\begin{cases} \sigma^{\nu\tau} =: \dfrac{i}{2}[\gamma^\nu, \gamma^\tau] \qquad \gamma^\nu: \text{Dirac matricies} \\ q^\nu = \underline{q} \\ F_1(q^2), F_2(q^2): \text{form factors} \quad \text{such that } F_1(0), F_2(0) = 1 \\ \kappa = g - 1 = 1{,}79 \quad (\text{proton}) \end{cases}$$

$$\longrightarrow \mathcal{M} \sim e[\bar{u}\gamma^\mu u]\left[\frac{-i g_{\mu\nu}}{q^2}\right] e\, \bar{u}\left[\gamma^\nu F_1 + \frac{i}{2} \frac{\sigma^{\nu\tau} q_\tau}{2M} \kappa F_2\right] u$$

$$\longrightarrow \frac{d\sigma}{d\Omega} \sim |M|^2 \cdot (\text{phase space})$$

$$\longrightarrow \boxed{\left.\frac{d\sigma}{d\Omega}\right|_{\text{Rosenbluth}} = (\dots) \frac{\alpha^2}{q^4} E'^2 \cos^2\frac{\theta}{2} \frac{E'}{E}\left[\left(F_1^2 + \frac{\kappa^2 Q^2}{4M} F_2^2\right) + \left(F_1 + \kappa F_2\right)^2 \frac{Q^2}{2M} \tan^2\frac{\theta}{2}\right]}$$

Or equivalently:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{Rosenbluth}} = (\dots)\,\frac{\alpha^2}{q^4}\,E'^2\,\frac{E'}{E}\left[\underbrace{\left(F_1^2 + \frac{K^2Q^2}{4M}F_2^2\right)}_{\sim\, a^2+b^2 \text{ term}} \times \cos^2\frac{\theta}{2} + \underbrace{\left(F_1 + KF_2\right)^2}_{\substack{\sim \text{ interference term} \\ (a^*b + ab^*)}}\frac{Q^2}{2M}\sin^2\frac{\theta}{2}\right]$$

In fact:

$$\sigma \propto |M|^2 = MM^\dagger \qquad \text{if } M = a+b \qquad M^\dagger = a^* + b^* \longrightarrow |M|^2 = a^2 + b^2 + a^*b + ab^*$$

Kinematic limits:

- $Q^2 \to 0$, $M = m_p \Rightarrow \dfrac{Q^2}{M^2} \longrightarrow 0$

$\frac{Q}{M}$ is the relative amount of energy that I'm exchange with target.

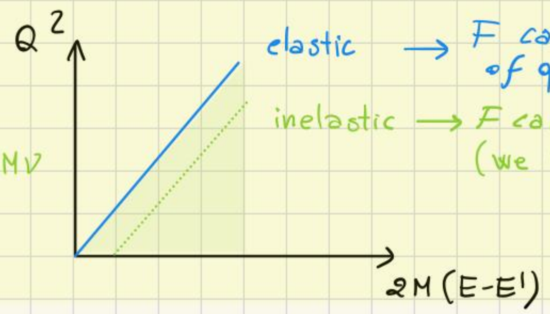
$$\longrightarrow \boxed{\left.\frac{d\sigma}{d\Omega}\right|_{\text{Rosenbluth}} \simeq \left.\frac{d\sigma}{d\Omega}\right|_{\substack{\text{Point}\\ \text{Like}}} \times |F_1(q^2)|^2}$$

OBSERVATION: Can $F(q^2)$ depends by anything else?

The spatial distribution $f(r)$ gives us $F(q^2)$. Can it depends by anything else?
We've seen that:

Elastic: $Q^2 = 2M\nu$

Inelastic: $W^2 = Q^2 - M^2 + 2M\nu$



elastic $\longrightarrow$ F can be function only of q^2: there are no others d.o.f.

inelastic $\longrightarrow$ F can be function also by something else (we have 2 Kinematic variables)

Form Factor for protons McAllister, Hofstadter 1956 Nobel Prize 1961

PHYSICAL REVIEW VOLUME 102, NUMBER 3 MAY 1, 1956

Elastic Scattering of 188-Mev Electrons from the Proton and the Alpha Particle*†‡§‖¶

R. W. McAllister and R. Hofstadter
Department of Physics and High-Energy Physics Laboratory, Stanford University, Stanford, California

(Received January 25, 1956)

The elastic scattering of 188-Mev electrons from gaseous targets of hydrogen and helium has been studied. Elastic profiles have been obtained at laboratory angles between 35° and 138°. The areas under such curves, within energy limits of ±1.5 Mev of the peak, have been measured and the results plotted against angle. In the case of hydrogen, a comparison has been made with the theoretical predictions of the Mott formula for elastic scattering and also with a modified Mott formula (due to Rosenbluth) taking into account both the anomalous magnetic moment of the proton and a finite size effect. The comparison shows that a finite size of the proton will account for the results and the present experiment fixes this size. The root-mean-square radii of charge and magnetic moment are each $(0.74\pm0.24)\times10^{-13}$ cm. In obtaining these results it is assumed that the usual laws of electromagnetic interaction and the Coulomb law are valid at distances less than 10^{-13} cm and that the charge and moment radii are equal. In helium, large effects of the finite size of the alpha-particle are observed and the rms radius of the alpha particle is found to be $(1.6\pm0.1)\times10^{-13}$ cm.

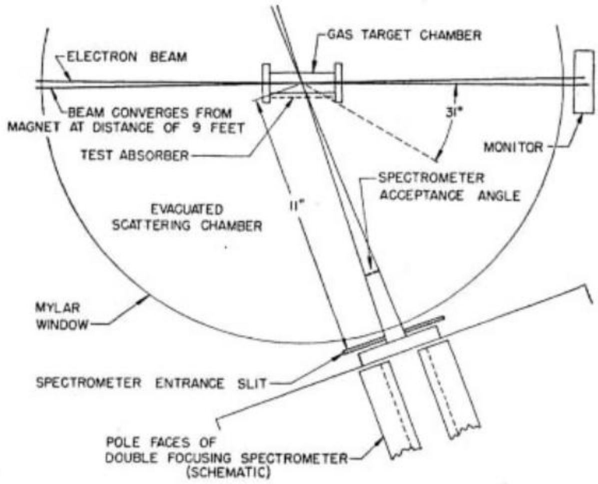


Fig. 2. Arrangement of parts in experiments on electron scattering from a gas target.

$$\boxed{e^- + H_2, He \longrightarrow e^- + X}$$

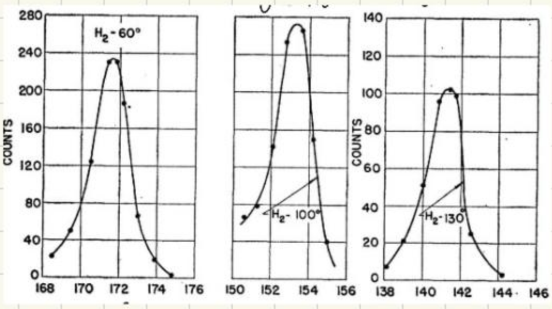
e^- beam 188 MeV

easy to do liquid Hidrogen

Measurement process: measure E' for $\theta \in [35, 138]^\circ$ and count # of particles

Theoretical aspect: $E' = \dfrac{E}{1 + 2\frac{E}{M}\sin^2\frac{\theta}{2}}$

60° 100° 130°

* 

→ Very narrow : res = ± 1,5 MeV (good approx for E')

Observations:

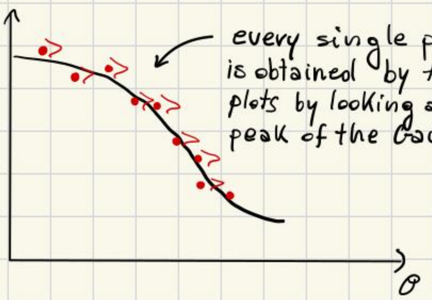
1) How can we verify that those are the expected behaviour? Let's take $\theta \simeq 60°$, $E = 200$ MeV, $M \simeq 1000$ MeV. Therefore:

$$\frac{E'}{E} \simeq \frac{1}{1 + \frac{E}{M}\left(\frac{1}{2}\right)^2} \simeq \frac{1}{1 + \frac{1}{20}} \simeq 1 - \frac{1}{20} = 1 - 0{,}05 \longrightarrow \boxed{E' \simeq 178 \text{ MeV}} \checkmark$$

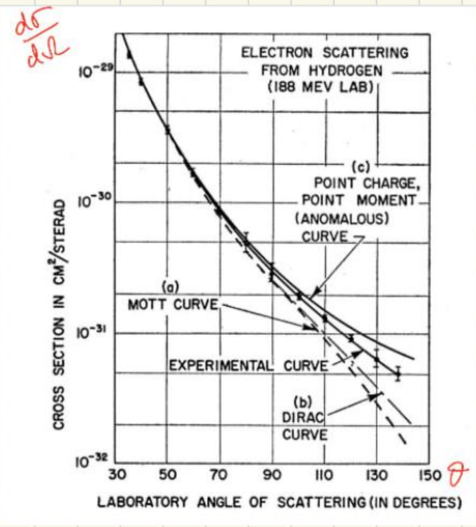
2) Increasing the angle, the energy decreases because of the dependence $E' \sim \dfrac{1}{1+\sin^2\frac{\theta}{2}}$

3) Also the number of events is affected by θ because of $\dfrac{d\sigma}{d\Omega} \propto \cos^2\dfrac{\theta}{2}$

4) If we do this for many θs we expect that $E' = \dfrac{E}{1 + 2\frac{E}{M}\sin^2\frac{\theta}{2}}$



every single point is obtained by the plots by looking at the peak of the Gaussians *



a) no structure $F_1 = 0$, no spin $F_2 = 0$ (MoH)

b) no structure $F_1 = 0$, with spin $F_2 \neq 0$, $g = 2$

c) no structure $F_1 = 0$, with spin $F_2 \neq 0$, $g \neq 2$ anomalous

What we measure does not agree with any of these curves., it's below (c). This is telling us that must be a structure infact if we modify (c) with the form factor (by def < 1) the new curve goes below (c) and matches the data point.

$\rightarrow F(q^2) \neq 0$

Therefore doing the expansion we can set, $F(q^2) \simeq 1 - \frac{1}{6} q^2 \langle r^2 \rangle$ and fit the data, estimating the best value for $\langle r^2 \rangle$.

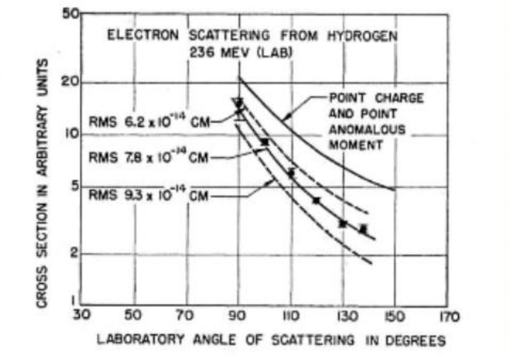


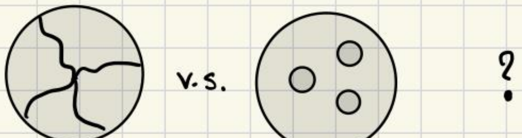
FIG. 6. This figure shows the experimental points at 236 Mev and the attempts to fit the shape of the experimental curve. The best fit lies near 0.78×10^{-13} cm.

$\longrightarrow$ $\boxed{\sqrt{\langle r^2 \rangle} = 0{,}78 \cdot 10^{-13}\,cm = 0{,}78\,fm}$

This is a prove that the proton is not point like moreover it is quite similar to $r_0 = 1{,}2\,fm$ (from $R_N = r_0 A^{1/3}$)

It is also a prove of the uncertainlly principle i.e. we need an energy $E \sim 200$ MeV to infer the structure of the proton.

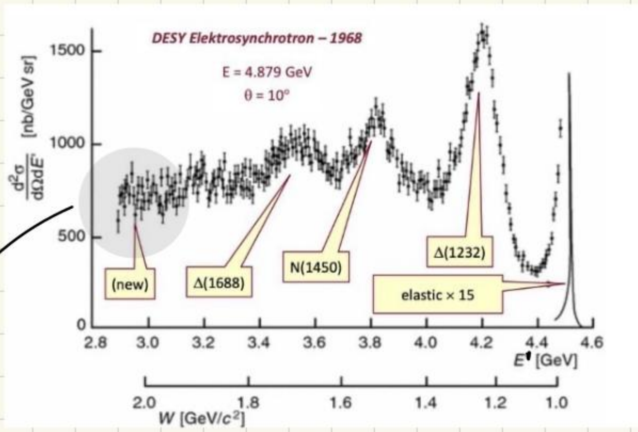
We've just proven that the proton has a structure. Now we need to understand how is it made internally:



To answer to this question we need to increase energy:

The 1st experiment with great update to the energy e-beam is:

$E_{beam} \simeq 5$ GeV
$\theta = 10°$



super inelastic regime: $e^- + p \longrightarrow e^- + X$ $W^2 = M^2 - Q^2 + 2M\nu$

The "inelastic" cross section can be written as:

$$\left.\frac{d\sigma}{d\Omega}\right|_{Inelastic} \simeq \frac{\alpha^2}{q^4} E'^2 \cos^2\frac{\theta}{2} \left[W_2(Q^2,\nu) + 2 W_1(Q^2,\nu) \tan^2\frac{\theta}{2} \right]$$

structure functions
they express ignorance about proton structure
(to measure exp)

Observations:

1) W_1 and W_2 need to be pure numbers because $[\sigma] = L^2 = E^{-2}$ $\left[\frac{E'^2}{q^4}\right] = E^{-2}$

2) We can also express all in terms of the Bjorken variables. In fact from an exp point of view measuring adimensional quantities is better:

$$(E',\theta) \longrightarrow (Q^2,\nu) \longrightarrow (x,y) \quad \text{where } x=\frac{Q^2}{2M\nu}\ ;\ y=\frac{\nu}{E}$$

and define:

$$\boxed{2M\,W_1(Q^2,\nu) =: F_1(x,y)} \qquad \boxed{\nu W_2(Q^2,\nu) =: F_2(x,y)}$$

Bjorken suggested that in principle:

$$W_2 = \frac{1}{\nu}F_2(Q^2,\nu) \xrightarrow{\text{suggest}} W_2 = \frac{1}{\nu}F_2\left(\frac{1}{x}\right)$$

where x is called Bjorken scaling variable.

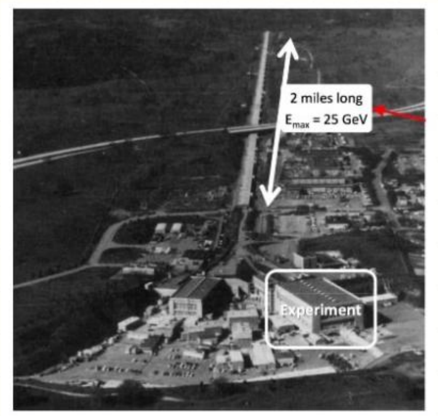
3) Comparing W_2 with W_1 from theory it should give us:

$$\frac{W_2}{W_1} = \left(\frac{Q^2}{\nu^2+Q^2}\right)(1+R) \qquad \text{where } R=\frac{\sigma_S}{\sigma_T}$$

σ_S → cross section for long. pol. γ^* → it can have longitudinal pol. because it is virtual i.e. it has a mass $\neq 0$ $m_{\gamma^*}=\sqrt{q^2}\neq 0$

σ_T → cross section for tranverse pol. γ^*

Experiment @ SLAC. e^- beam of 25 GeV v.s. protons.



With an energy of 25 GeV we can probe a spatial distance $\delta x \ll 1\,fm$.

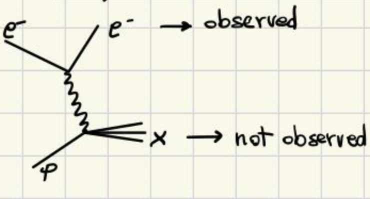
Remember that: $1\,fm = 200\ MeV^{-1} = 0{,}2\ GeV^{-1}$

→ We potentially could see the internal structure of p.

Goal: measure $\left.\frac{d\sigma}{d\Omega}\right|_{inel.}$ and compare with $\left.\frac{d\sigma}{d\Omega}\right|_{el}$

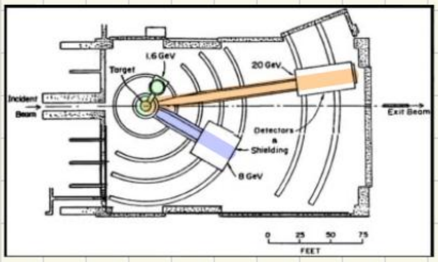
As we can see from the picture rails were built that were used to move the spectrometer in different positions

The spectrometer only measured electrons



- The target was liquid Hidrogen (or Helium)
- By changing the position of the spectrometer we changed θ and therefore it was possible, by counting the # of events, to obtain the cross section (the solid angle was known)
- By changing the magnetic field we change the energy of the beam

- There were 3 spectrometers :



  - one to measure energies of ≤ 1.6 GeV at $\theta \simeq 34°$
  - one to measure energies of ≤ 8 GeV
  - one to measure energies of ≤ 20 GeV

  } at $\theta \geq 12°$

- With the 1st detector we are able to count the number of times we see an <u>elastic scattering</u>. Doing that we can monitor 2 things:

  - The uniformity of the liquid target: if the rate drops this means that the density of the target drops (this is due by the fact that we can destroy the target in inelastic collisions)

  - The luminosity of the machine : # el events $\equiv N_{el} = \mathcal{L} \cdot \sigma_{el}^{th} \longrightarrow \mathcal{L} = \frac{N_{el}}{\sigma_{el}^{th}}$

- With the others 2 spectrometers we are able to see <u>inelastic scattering</u>

The schematics of the 3 spectrometers is the following :

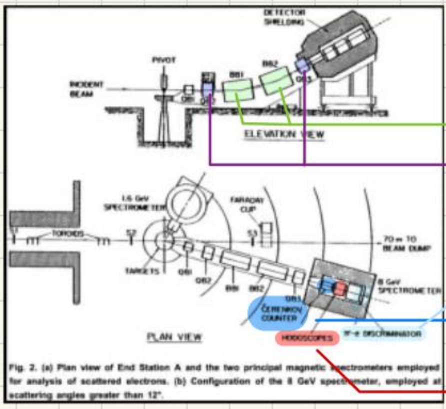


Fig. 2. (a) Plan view of End Station A and the two principal magnetic spectrometers employed for analysis of scattered electrons. (b) Configuration of the 8 GeV spectrometer, employed at scattering angles greater than 12°.

→ dipole magnets (to bend the beam) (useful to reduce b.g.)

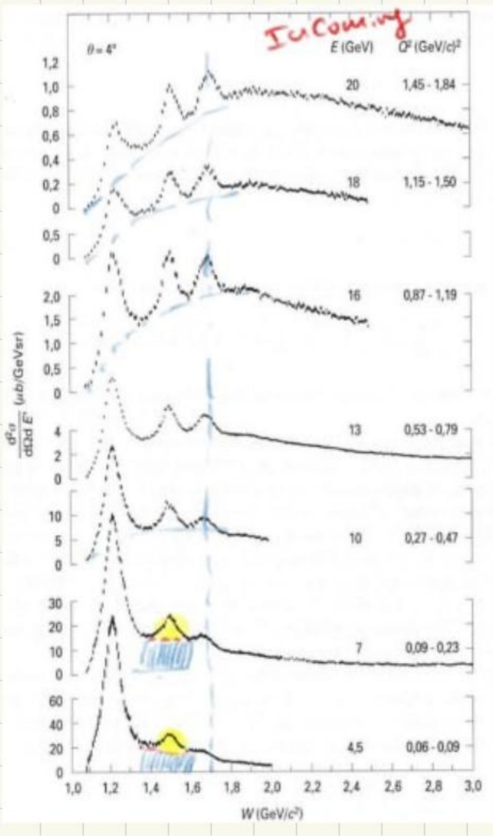
→ quadrupole magnets (to focus the beam)

→ shower counter (we discriminate e^-/π^- via shower properties)

→ the Cherenkov counter is useful to discriminate e^- and π

→ the Hodoscope (a thin scintillation plastic (or glass) layer is useful to know if something goes through

The outcome of the experiment is the following :



On the orizontal axis there is W :

$$\boxed{W^2 = M^2 - Q^2 + 2M\nu} \quad \text{where } Q^2 = 4EE'\sin^2\frac{\theta}{2}$$

(E is fixed by the beam, $\theta = 4°$, E' is computed from E $\longrightarrow$ we can compute Q^2 and then W^2)

On the vertical axis there is the double differential cross-section :

$$\boxed{\frac{d^2\sigma}{d\Omega\, dE'} = \frac{\#\,\text{events}}{(\Delta\Omega)(\Delta E')}} \quad \left(\frac{\mu b}{GeV \cdot sr}\right)$$

1) Increasing energy, Q^2 increases, the highest peak (elastic peak) decreases while the resonance peaks (inelastic peaks) become greater (the yellow area becomes bigger)

2) At fixed W σ decreases if E increases (expected because $\sigma \propto \frac{1}{s}$)

3) At fixed E and θ the resonances decrease if W increases (and this makes sense because if the resonance is heavier it needs more energy)

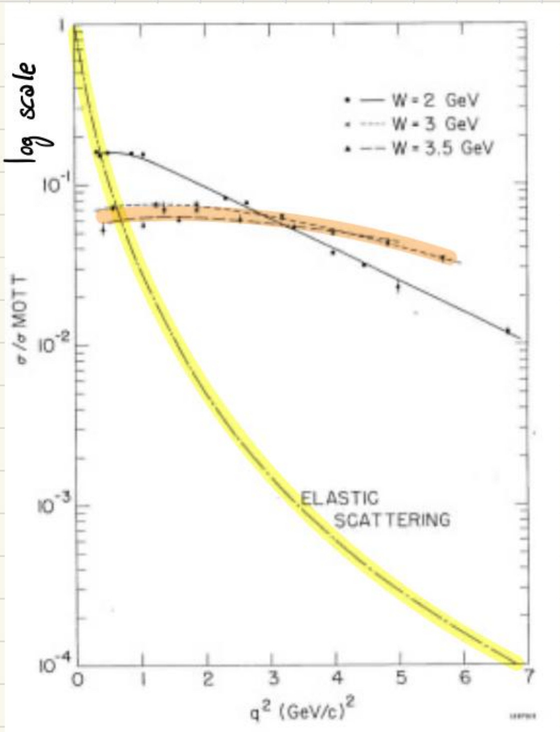
4) For very large W: $\frac{d\sigma}{d\Omega\, dE'} \simeq 1-2 \frac{\mu b}{GeV \cdot sr}$ (plateau)

So, we know that with D.I.S. we have:

$$\boxed{\frac{\left.\frac{d\sigma}{d\Omega}\right|_{DIS}}{\left.\frac{d\sigma}{d\Omega}\right|_{Mott}} = W_2(Q^2,\nu) + 2\, W_1(Q^2,\nu)\tan^2\frac{\theta}{2} \overset{\text{fixed } \theta}{\equiv} R(Q^2)}$$

where $\left.\frac{d\sigma}{d\Omega}\right|_{Mott} = \frac{\alpha^2}{Q^4} E'^2 \cos^2\frac{\theta}{2} \frac{E'}{E}$ (Mott with recoil)

What they saw is the following:



Here the angle is fixed at $\theta = 10°$

In this plot we are able to see the form factor for the scattering against a proton. That is because when dividing by σ_{Mott} we take away all the kinematics

We highlight the elastic case in the pointlike hyp.: at $R(Q^2=0)=1$ and then it decreases. In fact:

$$\frac{\left.\frac{d\sigma}{d\Omega}\right|_{elastic}}{\left.\frac{d\sigma}{d\Omega}\right|_{Mott}} = \left(1 - \frac{q^2}{4M^2}\tan^2\frac{\theta}{2}\right) \underset{1 \text{ (point like)}}{\underbrace{|F(q^2)|^2}}$$

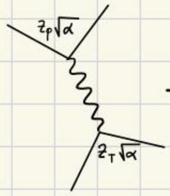
From the inelastic measures instead we do not see any q^2 dependence $\sim$ flat.

Considering the proton with a structure we know that the only modification is:

$$\left.\frac{d\sigma}{d\Omega}\right|_{Struct} = \left.\frac{d\sigma}{d\Omega}\right|_{Point\ Like} \cdot |F(q^2)|^2$$

from the measures we deduce that $F(q^2)$ has not dependence from q^2 and this would mean a point like particle. However the results that we have are not compatible with the pointlike scattering and moreover its value is $\neq 1$.

Maybe this behaviour is due by the



$\longrightarrow \sigma \; \dfrac{(Z_p Z_T \alpha)^2}{Q^4} \overset{e^-:\, Z_p = 1}{\Longrightarrow} \sigma = Z_T^2 \dfrac{\alpha^2}{Q^4}$

This means that: $\dfrac{\sigma}{\sigma_{Mott}} = Z_T^2 < 1 \longrightarrow \boxed{Z_T < 1}$

So we see a reduced cross section because the charge of the target is not 1 but lesser.

The next thing that they did was the measurement of $F_2(Q^2,\nu)$ v.s. $\omega = \frac{2M\nu}{Q^2} = \frac{1}{x}$

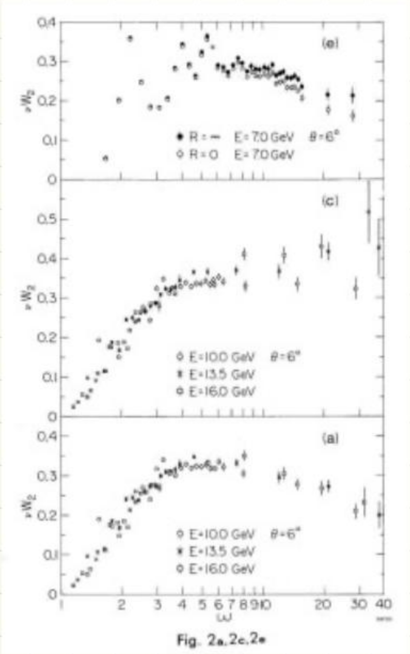


Fig. 2a,2c,2e

We are focusing on $F_2(Q^2,\nu) = \nu \cdot W_2(Q^2,\nu)$ because for small angle $\frac{d\sigma}{d\Omega}$ is more sensitive to W_2

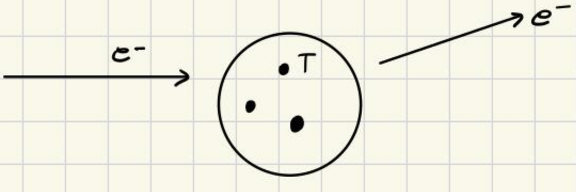
We see that F_2 (W_2) is sensitive to $\frac{\nu}{Q^2}$ not to Q^2 and after a certain ω the curve is $\sim$ flat.

Therefore this experiment managed to prove that with high energy collisions we are probing the pieces inside the proton

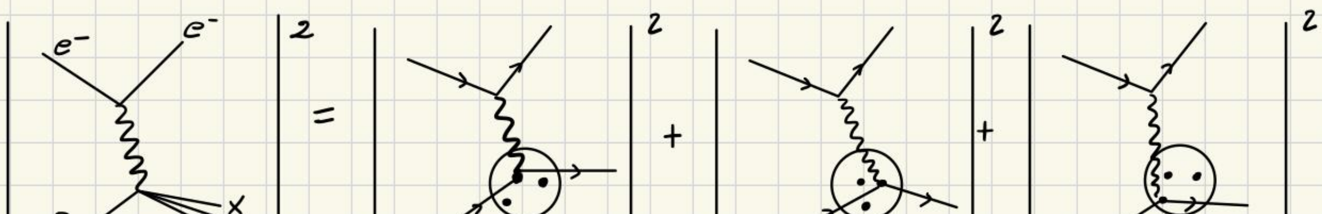
There is a calculation done by Feynman that shows that the x of Bjorken is just the fraction of momentum taken away by each piece inside the proton

QUARK PARTON MODEL

This tells us that maybe proton is a container of smaller particles with fractional charge (not multiple of e) and $\frac{1}{2}$ spin (we have to prove it). These particles are called partons and are the building blocks of the QUARK PARTONS MODEL



In particolar the idea is that D.I.S. is an incoherent superposition of elastic scattering with partons: (the number of partons from data = 3)



Therefore the assumption is that proton exchanges γ^* with 1 parton at the time

What we want to do is to translate the variables which we experimentally measure in a DIS into variables (statistical functions) that describe the single partons inside the proton looking at DIS ($e^- p$) we saw that:

$$\left.\frac{d^2\sigma}{d\Omega\, dE'}\right|_{DIS} = \frac{\alpha^2}{Q^4} E'^2 \left[W_2(Q^2,\nu) \cos^2\frac{\theta}{2} + 2\, W_1(Q^2,\nu) \sin^2\frac{\theta}{2} \right]$$

These W_1 and W_2 are the structure functions: they model our ignorance on what happens at the quark level (Feynman and Bjorken tried to calculate them Nobel prite)

Let's look at a generic elastic scattering with a parton X with spin $1/2$, pointlike, with a mass m such that $m_x = M_p \cdot x$ ($0 < x < 1$) and with fractional charge Z_x:

$$\frac{d\sigma}{d\Omega} \propto \underbrace{\frac{\alpha^2}{Q^4} Z_q^2 E'^2}_{\text{Mott}} \underbrace{\frac{E'}{E}}_{\text{recoil}} \underbrace{\cos^2\frac{\theta}{2}}_{s=1/2 \text{ probe}} \left[1 + \frac{Q^2}{2M^2} \tan^2\frac{\theta}{2} \right] \delta\left(\nu - \frac{Q^2}{2m_x}\right)$$

→ conservation of energy for partons

$s = 1/2$ target, $g = 2$ } Dirac point like $s = 1/2$ particle target

$\delta\left(\nu - \frac{Q^2}{2m_x}\right) \rightarrow \nu = \frac{Q^2}{2m_x}$

$\rightarrow Q^2 = 2 m_x \nu$ (elastic case)

$\rightarrow Q^2 = 2 x M_p \nu \rightarrow x = \frac{Q^2}{2M_p\nu}$ ✓

This implies that

$$\frac{d\sigma}{d\Omega\, dE'} = (---) E'^2 \left[\cos^2\frac{\theta}{2} + \frac{Q^2}{2m^2} \sin^2\frac{\theta}{2} \right] \delta\left(\nu - \frac{Q^2}{2m}\right)$$

Therefore we have that:

$$\boxed{\begin{aligned} W_1(Q^2,\nu) &= \frac{Q^2}{4m^2}\, \delta\left(\nu - \frac{Q^2}{2m}\right) Z_q^2 \\ W_2(Q^2,\nu) &= Z_q^2\, \delta\left(\nu - \frac{Q^2}{2m}\right) \end{aligned}}$$ for one parton with x

If we define the parton density function $f(x)$ such that:

$f(x)\, dx$ = prob. to find a parto with fraction of mass $\in [x, x+dx]$ $\int_0^1 f(x)\, dx = 1$

Therefore we could take into account all the values of x:

$$W_1(Q^2,\nu) = \int_0^1 dx\, f(x) \cdot \frac{Q^2}{4m^2} Z_q^2\, \delta\left(\nu - \frac{Q^2}{2m_x}\right) =$$

$$= \int_0^1 dx\, f(x) \cdot \frac{Q^2}{4M^2x^2} Z_q^2\, \delta\left(\nu - \frac{Q^2}{2M\cdot x}\right)$$

We use that $I = \int A(x)\, \delta(g(x))\, dx = \frac{A(x_0)}{|g'(x_0)|}$ where x_0 is such that $g(x_0) = 0$

In our case $A(x) = \frac{f(x)\, Q^2}{x^2 \cdot 4M^2} Z_q^2$; $g(x) = \nu - \frac{Q^2}{2Mx}$; $g'(x) = \frac{Q^2}{2Mx^2}$; $g(x=x_0) = 0 \leftrightarrow x_0 = \frac{Q^2}{2M\nu}$

$\longrightarrow \left. g'(x)\right|_{x_0} = \frac{Q^2}{2M}\frac{1}{x_0^2}$ $\longrightarrow I = Z_q^2 \left.\frac{Q^2 f(x)}{4M^2 \cdot x^2}\right|_{x_0} \cdot \frac{2M}{Q^2} x_0^2 = f(x_0) \cdot \frac{Z_q^2}{2M}$

$$\longrightarrow \boxed{W_1(Q^2,\nu) = Z_q^2 \left.\frac{f(x)}{2M}\right|_{x=x_0}}$$

In analogy:

$$\boxed{W_2(Q^2,\nu) = Z_q^2 \frac{f(x)\, x}{\nu}\Big|_{x=x_0}}$$

For N partons we generalize as:

$$\boxed{\begin{aligned} W_1(Q^2,\nu) &= \sum_j^N Z_{q_j}^2 \frac{f_j(x)}{2M} \\ W_2(Q^2,\nu) &= \sum_j^N Z_{q_j}^2 f_j(x) \frac{x}{\nu} \end{aligned}}$$

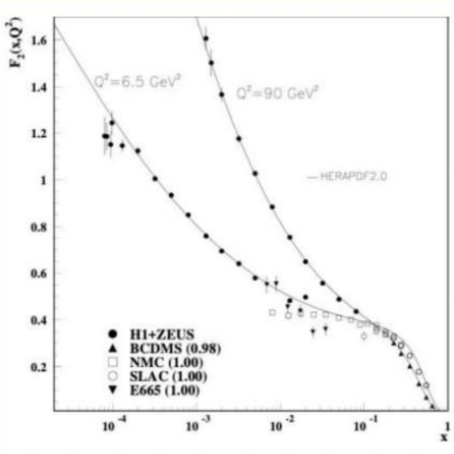
In the literature is also used the notation

$$\boxed{\begin{aligned} F_1(x) &\equiv M W_1 \;; \\ F_2(x) &\equiv \nu W_2 \end{aligned}}$$

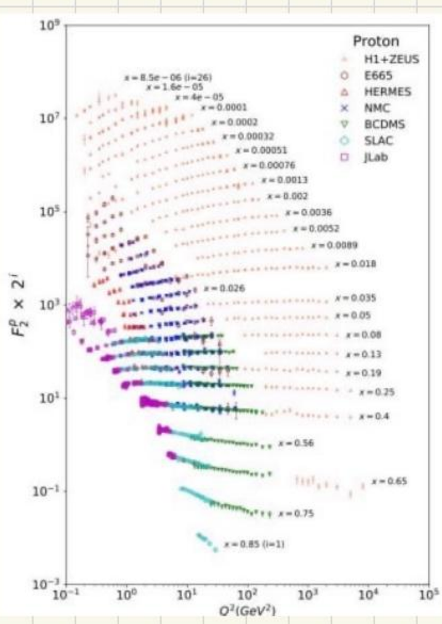
we then have that:

$$\boxed{F_2(x) = 2x\, F_1(x)}$$ Callan - Gross relation

this relation tells us that we have one structure function not 2 because they are correlated. This helped also the experimentalist since now only the measurement of 1 function is needed



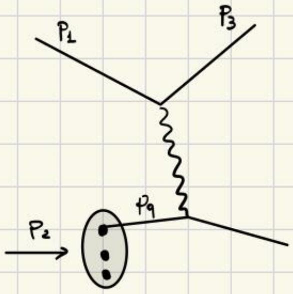
- In the plot just F_2 is shown
- The measurements have been done at fixed value of Q^2 (that we fix by fixing the beam energy E and the deflection angle θ.



- Here is shown the Bjorken scaling.
- All the measurements were done with $e^- + p \to e^- + X$ (and that is the reason of p on top of F_2). This is because in principle the p.d.f. could be different for different nucleons. Today we know instead that these p.d.f. are pretty universal.
- In this plot we can clearly see the Bjorken scaling: for a fixed x the behaviour as a function of Q^2 is flat. The dipendence from x instead is more evident

Feynman point of view : x : property of parton

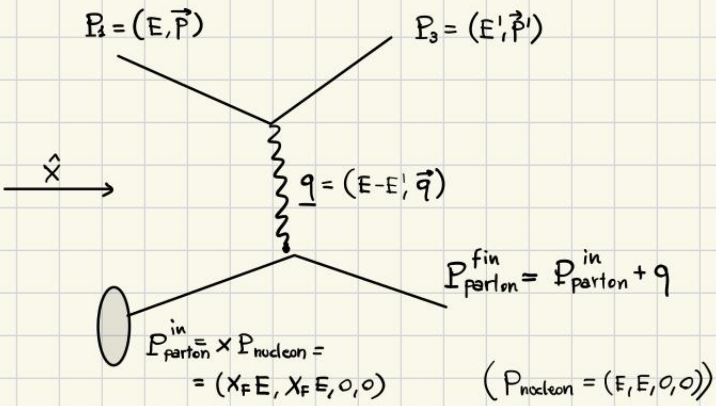
His idea is that the initial momentum of the parton is a fraction of the momentum of the nucleon.



$$P_q = x_F P_2 \longrightarrow \boxed{x_F = \frac{|\vec{P}_{parton}|}{|\vec{P}_{nucleon}|}} \approx \frac{|\vec{P}_L|}{|\vec{P}_{nucleon}|}$$

N.B. the transverse momentum of the parton is negligible because the Fermi energy is $E_{Fermi} = 200$ MeV $\longrightarrow P_{Fermi} = 40\text{-}45$ MeV while we now have $\sqrt{s} \simeq 20$ GeV that is quite higher.

There is a relation between the Bjorken variable x and the Feynman x and this can be seen in the high energy limit. $m_e, m_p, m_{parton} \ll E$



$$P_{part}^{fin} = P_{part}^{in} + q \longrightarrow |P_{part}^{fin}|^2 = |P_{part}^{in}|^2 + |q|^2 + 2P_{part}^{in} \cdot q$$

$$\left(\text{but } |P_{part}^{fin}|^2 = |P_{part}^{in}|^2 = m_{part}^2 \ , \ |q|^2 = -Q^2\right)$$

$$\longrightarrow 0 = -Q^2 + 2P_{part}^{in} \cdot q$$

($P_{part}^{in} \cdot q$ is a Lorentz invariant and I can compute it in the frame where the parton is at rest : $2P_{part}^{in} \cdot q = 2(x_F M, 0, 0, 0)(E-E', \vec{P}-\vec{P}^{in})$
$= 2x_F M(E-E') = 2x_F M \nu$)

$$\longrightarrow Q^2 = 2P_{part}^{in} \cdot q = 2x_F M \nu$$

$$\longrightarrow \boxed{x_F = \frac{Q^2}{2M\nu}}$$

So, in the high energy limit x_F and x_B are the same thing and this is an universal value. So we're able to extract infos about the single parton : what we have to do is to measure $\frac{d^2\sigma}{d\Omega dE}$ and from this determine $F_2(x) = x F_1(x)$

Now we have to prove how many partons there are

With N partons we have N fractions of momentum x_i ; $i = 1, \dots, N$ $\quad x_i \in [0, 1]$ that are such that :

$$\sum_{j=1}^{partons} x_j = 1 \longrightarrow \sum_i \underline{P}_{part}^{i} = \underline{P}_{nucleon}$$

Then we have additional constraints : describing the proton for example we should impose that the charge is $+1$ (and that is colorless).
In this model we define 3 types of partons :

1) VALENCE QUARKS
2) SEA OF QUARK - ANTIQUARK PAIRS
3) SEA OF GLUONS : mediators of strong interaction

- The Valence Quarks are those such that the charge of the proton is correct and the same for the other quantum numbers.

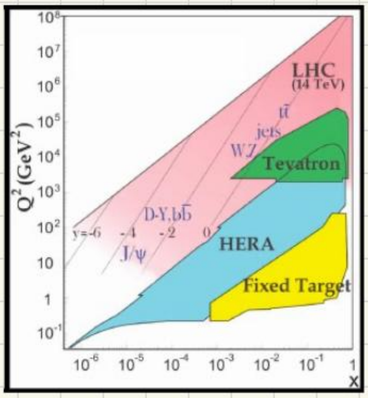
- Then we could add $q\bar{q}$ pairs ($q\bar{q}$ sea) and gluons (g sea) without any problem and write the proton like

$$p = \underbrace{uud}_{\text{valence quarks}}\ \underbrace{u\bar{u}\,d\bar{d}\,s\bar{s}}_{q\bar{q}\text{ sea}}\ \underbrace{g}_{g\text{ sea}}$$

So the nucleon is a container of many quarks exchanging gluons. What we find experimentally is that

$$\sum_j^{\text{valence}} x_j = 0.5$$

and this means that only half of the quarks is due to valence quarks and all the other mess is due to the sea of quarks and gluons. In order to obtain this value was sum all the momentum of the quarks (reconstructed from hadron jets) and see that they add up only to half of the proton momentum. In order to have mass conservation these particles in an infinite see of quarks – antiquarks and gluons must have a very small x.
To being able to probe these small values of x we need high energies.

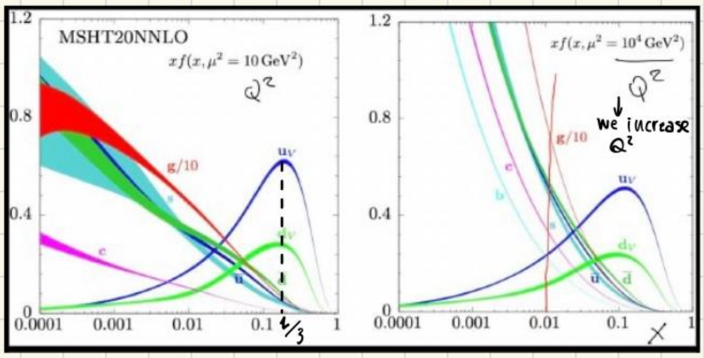


- In the 70s we started with fixed target experiments (e.g. SLAC), then came HERA with e^- beam against a proton beam, then TEVATRON with a c.o.m. energy of 2 TeV and today LHC with 14 TeV.

So if partons exist we have to be able to see them, but what? We have to measure the partons density functions.

$$\boxed{\begin{array}{ll} f_v(x) & \text{for the valence quarks} \\ f_s(x) & \text{for the sea quarks} \\ f_{\bar{s}}(x) & \text{for the sea antiquarks} \\ g(x) & \text{for the sea gluons} \end{array}}$$

From LHC we have the following plots:



- the width of the lines is the experimental uncertainty.

- from the plot we can see that as expected the u quark takes about $1/3$ of proton momentum

- In the plot is shown $g/10$ because at LHC the $p+p$ collision is mainly $g+g$.

- From the plot we can also see that if we put ourself at very small x we have more gluons than quarks

- Increasing Q^2 is more likely to find the sea even at large x.

Constraints on the parton density functions in the case of the proton

- $\sum_i \underline{P}^i_{part} = \underline{P}_{nucleon} \longrightarrow \int_0^1 dx \left[\sum_i^{\text{all partons}} f_i(x)\right]\cdot x = 1 \longrightarrow \boxed{\int_0^1 dx \left[\sum_{flavor} q_f(x) + \bar{q}_f(x) + g(x)\right]\cdot x = 1}$ Momentum sum rule

- $\int_0^1 dx \left[u_v^p - \bar{u}_s^p\right] = 2$ 2 valence up quarks in total

- $\int_0^1 dx \left[d^p - \bar{d}^p\right] = 1$ 1 valence down quark in total

(the last two: Baryon number sum rule)

Does the parton density function depend on the nucleon?

For example $u^p(x) = \text{or} \neq u^n(x)$? We know that $p = (uud)$; $n = (udd)$. In principle should depend on the nucleon but in practice the do not depend on them. The parton density functions are pretty universal).

Evolution of parton density function

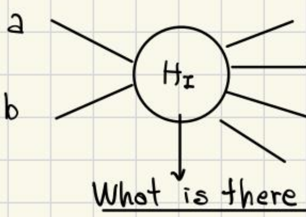
The DGLAP theory gives a way to predict the evolution of the parton density function for a given measured value of it at a given starting point Q^2.

In the 1950-1960 the known particles were $p, n, \pi^\pm, K^\pm, \pi^0, K^0, \Delta, \dots$. Once the proton was discovered to be not a pointlike particle anymore, the question was what are the fundamental particles? And how they build also the other know particles?
We'll see that :

Baryons : $q_1 q_2 q_3$

Mesons : $q_1 \bar{q}_2$

In order to understand that we always have to study something like:



events made of hadrons / leptons $\propto \sigma \propto |M|^2 \rho(E_f)$ where $M = \langle f|H_I|i\rangle$

What is there in this mess ?

First of all in order to study H_I we should study the symmetries of the theory since they can provide informations about H_I without knowing it (they can exclude som final states providing some selection rules).

SYMMETRY : unitary / antiunitary operators

Noether's theorem : symmetry $\longleftrightarrow$ conserved quantity

Constant of motion : it is a physical quantity only for hermitian operators.

External Symmetries : act on $\vec{x}$ and t

Internal Symmetries : act on internal q. numbers

Continuous $\quad \mathcal{U}(a_1, \dots, a_n) = e^{i\alpha_j T_j}$

T_j : generators of U : $T_j = T_j^{+}$ hermitian
α_j : parameters

Example $\quad \mathcal{U}(\vec{x}) = e^{i\vec{p}\cdot\vec{x}}$; $\mathcal{U}(t) = e^{iEt}$; $\mathcal{U}(\vec{\theta}) = e^{i\vec{L}\cdot\vec{\theta}}$

discrete transformation : e.g. $\mathbb{P}$, $\mathbb{C}$

$\mathbb{P}$ Parity :

- It's an inversion $\quad \mathbb{P} : \vec{x} \mapsto -\vec{x} \quad , \quad \vec{p} \longrightarrow -\vec{p}$

If we apply twice $\quad \mathbb{P}\mathbb{P}\psi = \psi$
$\mathbb{P}^{+} = \mathbb{P}$ $\quad$ (hermitian operator $\longrightarrow$ the eigenvalues have a phys. meaning
$\mathbb{P}^2 \psi = a^2 \psi \quad \longrightarrow \quad a^2 = 1 \longrightarrow a = \pm 1$

- Why is it so important : Parity is conserved in strong and EM interactions.

Intrinsic parity : eigenvalue parity in the particle rest frame . It's Lorentz invariant.
n.b. for massless particle we have to use Q.F.T.

$$\boxed{\begin{array}{l}
\cdot \text{ Leptons } (q=-1),\ e^-,\ \nu_e \quad P=+1 \\
\cdot \text{ Quarqs} \qquad\qquad\qquad\qquad P=+1
\end{array}\Bigg\}\ S = 1/2 \text{ fermions}}$$

- Anti-fermions $\quad P = -1$
- Anti-bosons : same parity as bosons
- $f\bar{f}$ pair : $P = -1$
- $B\bar{B}$ pair : $P = +1$
- γ : $P = -1$

$\mathbb{C}$ Parity

It is an inversion : $\mathbb{C}\mathbb{C}\psi = a\psi \quad \longrightarrow a = \pm 1$

$\mathbb{C}$: changes particle in its antiparticle ($\vec{x}, \vec{p}, \vec{s}$ are unchanged)

All quantum numbers : $(-1) \times$ (quantum number) $\xrightarrow{e.g.}$ $\mathbb{C}: q \longrightarrow -q$ electric charge

$\mathbb{C}\psi = a\psi$ only neutral particles can be eigenstates . n, γ, π^0, ν ?

γ is an eigenstates , $n \neq \bar{n}$, $\nu = \bar{\nu}$ (We have to see experimentally case to case)

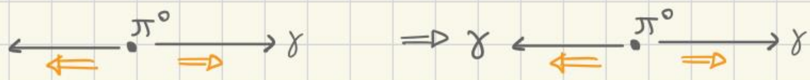
Why is it so important : Strong and E.M. interactions conserve $\mathbb{C}$ and it's interesting to study $\mathbb{C}$ acting on states of N particles

$\pi^+\pi^-$ $\quad\quad$ $\pi^-\pi^+$

$\pi^+\pi^-\pi^0$ $\quad\quad$ $\pi^-\pi^+\pi^0$

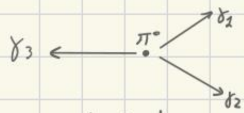
Examples:

- $\pi^0 \longrightarrow \gamma\gamma$ EM decay $\mathbb{C}$:



$$\mathbb{C}\psi_{\pi^0} = C_\gamma C_\gamma \psi_{\pi^0} \longrightarrow (C_\gamma)^2 = C_{\pi^0} \xrightarrow{C_\gamma = -1} C_{\pi^0} = +1$$

- $\pi^0 \longrightarrow \gamma\gamma\gamma$

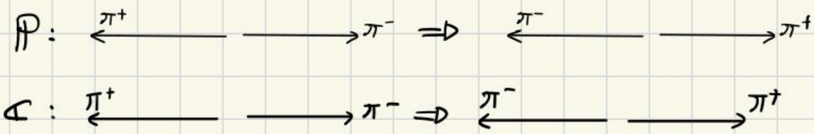


We know that $C_{\pi^0} = +1$ but here we get $C_{\pi^0} = (-1)^3 = -1$ NOT POSSIBLE : it is forbidden by $\mathbb{C}$ conservation in E.M. interactions

Is important to study parity for $B\bar{B}$ $(C=+1)$, $F\bar{F}$ $(C=-1)$

Two examples:

- $\pi^+\pi^-$ $\quad$ $\mathbb{C}: \pi^+\pi^- \longrightarrow \pi^-\pi^+$ $\quad$ $\mathbb{C}\psi_{\pi^+\pi^-} = a\,\psi_{\pi^+\pi^-}$

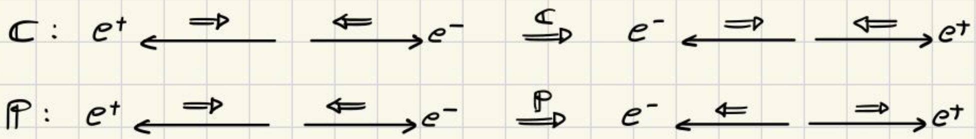


$\Rightarrow \mathbb{C}\psi_{\pi^+\pi^-} \equiv \mathbb{P}\psi_{\pi^+\pi^-} = (-1)^L \psi_{\pi^+\pi^-}$ $\quad$ L : angular momentum of $\pi^+\pi^-$ system

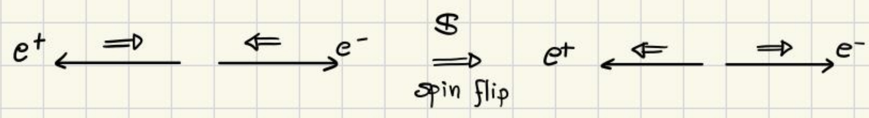
$C_{\pi^+\pi^-} = C_{\pi^+} C_{\pi^-} (-1)^L$ $\quad$ Since π are bosons $C_{\pi^+}C_{\pi^-} = +1$ $\longrightarrow C_{\pi^+\pi^-} = (-1)^L$

$$\longrightarrow \boxed{C_{B\bar{B}} = (-1)^L}$$

- e^+e^- $\quad$ $s = 1/2$ $\quad$ $\mathbb{C}: e^+ \longrightarrow e^-$



} they do not give the same result



$e^+ \xleftarrow{\Rightarrow} \quad \xleftarrow{\Leftarrow} e^- \quad \underset{\mathbb{P}+\mathbb{S}}{\Longrightarrow} \quad e^- \xleftarrow{\Rightarrow} \quad \xrightarrow{\Leftarrow} e^+$

Therefore doing a Charge conjugation is equal to apply Parity and SpinFlip.

$$\begin{cases} \mathbb{P}\,\psi_{e^+e^-} = (-1)^L \psi_{e^+e^-} \\ \mathbb{S}\,\psi_{e^+e^-} = \mathbb{S}\,\psi_{space}\psi_{spin} = \psi_{space}\,\mathbb{S}\,\psi_{spin} = (-1)^{S+1}\psi_{e^+e^-} \end{cases}$$

Recall: $\begin{cases} \mathbb{S}\,\psi_{B\bar{B}} = (-1)^S \psi_{B\bar{B}} \\ \mathbb{S}\,\psi_{F\bar{F}} = (-1)^{S+1}\psi_{F\bar{F}} \end{cases}$

$$\longrightarrow \mathbb{C}\,\psi_{e^+e^-} = (\mathbb{P}\cdot\mathbb{S})\,\psi_{e^+e^-} = (-1)^L\cdot(-1)^{S+1}\underbrace{C_{e^+}C_{e^-}}_{-1} = (-1)^{L+S}(-1)^2 = (-1)^{L+S}$$

$$\longrightarrow \begin{cases} \mathbb{C}\,\psi_{F\bar{F}} = (-1)^{L+S}\psi_{F\bar{F}} \\ \mathbb{C}\,\psi_{B\bar{B}} = (-1)^{L+S}\psi_{B\bar{B}} \quad \text{(We can check it)} \end{cases} \Longrightarrow \boxed{C_{p\bar{p}} = (-1)^{L+S}}$$

L: orbital ang. mom
s: spin of particle p

What happen to multiparticle system?

Suppose we have a system of more particles e.g. $\psi = \psi_1\psi_2$

- For continuous operators $\longrightarrow$ additive eigenvalues

Example: $\mathcal{U}(a) = e^{iaG}$ $\quad a \to 0 \quad \mathcal{U} = \mathbb{1} + iaG$

$$\longrightarrow \mathcal{U}\psi_1\psi_2 = (\mathcal{U}\psi_1)(\mathcal{U}\psi_2) = (\mathbb{1} + iaG)\psi(\mathbb{1} + iaG)\psi_2 \qquad G\psi_i = g\psi_i$$

$$= (1 + iag_1)\psi_1(1 + iag_2)\psi_2 \sim e^{i(g_1+g_2)a}\psi_1\psi_2 \longrightarrow g_{12} = g_1 + g_2$$

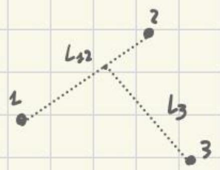
- For discrete transformation $\longrightarrow$ multiplicative eigenvalues

Parity:

- $\psi = \psi_1\psi_2$ $\qquad \mathbb{P}\,\psi = P_1P_2\,(-1)^{\ell}\psi_{space}$
  - pair fermion - antifermion : $P = P_1P_2(-1)^L = (-1)^{L+1}$
  - pair boson - antiboson : $P = (P_1P_2)(-1)^L = (-1)^L$

- $\pi^+ = u\bar{d} \longrightarrow P_{\pi^+} = (+1)(-1)(-1)^0 = -1$ $\qquad \mathbb{P}\;\; \xrightarrow{\pi^+} \;\Longrightarrow\; \xleftarrow{\pi^+} \qquad \mathbb{P}|\pi^+\rangle = -|\pi^+\rangle$

- $\psi = \psi_1\psi_2\psi_3 \longrightarrow \mathbb{P}\psi = P_1P_2P_3\,\mathbb{P}\,\psi_{space}$



L_3: mom. of 3 w.r.t. center of mass of 12

$$P = P_1P_2P_3(-1)^{L_{12}+L_3}$$

- proton: $uud \longrightarrow P = (+1)(+1)(+1)(-1)^0 = +1$ ($L=0$, no excited state)
- neutron: $udd \longrightarrow P = (+1)(+1)(+1)(-1)^0 = +1$

ISOSPIN : Continuous internal symmetry

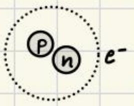
From an hystorical point of view Heisemberg noticed that $m_p \simeq m_n$ $\Delta m \simeq 2\,\text{MeV}$ $(\Delta m / m \sim 2/1000)$ not an accident.

Nucleon $= \begin{pmatrix} p \\ n \end{pmatrix}$ doublet of Isospin I (same algebra as $\vec{J}$)

$p = |I = 1/2, I_3 = +1/2\rangle$ $\quad n = |I = 1/2, I_3 = -1/2\rangle$

$\pi^+ \pi^-$ have same mass : $m_{\pi^0} \simeq m_{\pi^+}$ $\quad \frac{\Delta m_\pi}{m_\pi} \simeq \frac{5}{140}$

Pion triplet : $\pi^+ = |1, +1\rangle$ $\quad \pi^- = |1, -1\rangle$ $\quad \pi^0 = |1, 0\rangle$

Deuterium : isotope of H 

; deuton : nucleus of deuterium (seen via $\pi^+ p \to d\pi^0$)

$nn = ?$
$pn = ?$
$pp = ?$

$$p \otimes n = |1/2\ 1/2\rangle \otimes |1/2\ -1/2\rangle \begin{cases} \nearrow pn \text{ singlet } |0,0\rangle \\ \searrow pn \text{ triplet } |1,0\rangle \end{cases}$$

We don't see bound pp or nn $\longrightarrow$ hypothesis pn is isospin singlet

ISOSPIN

When there start to be more than say 15 different fundamental particles you start thinking that they're not really fundamental but there is something more fondamental at a deeper level. That is why we need to study ISOSPIN.

Heisemberg came up with this idea looking at n and p: they have more or less the same mass $\Delta m \sim 2$ MeV $\rightarrow \frac{\Delta m}{m} \sim \frac{2}{1000}$. The idea was that they are different states of the same isospin particle.

$$\text{nucleon} = \begin{pmatrix} p \\ n \end{pmatrix} \quad , \quad I = \frac{1}{2} \quad , \quad I_3 = \pm\frac{1}{2}$$

Same thing happens for $\pi^\pm$ and π^0 $\frac{\Delta m}{m} \sim \frac{3 \text{ MeV}}{140 \text{ MeV}} \longrightarrow \begin{pmatrix} \pi^+ \\ \pi^- \\ \pi^0 \end{pmatrix}$ $I = 1$; $I_3 = \pm 1, 0$

So the idea was that the strong int. maybe has a symm. such that different states of the same isospin behave in the same way under the interaction.

- The 1st experimental evidence came from deuton (pn) that is the nucleus of deuterium ($Z=1$, $A=2$). We obtain it by exciting an atom of H and then remove the electron with an $\vec{E}$. So Heisemberg though that if pn exists maybe we also have pp or nn:

Knowing that $|p\rangle = |\frac{1}{2}, \frac{1}{2}\rangle$, $|n\rangle = |\frac{1}{2}, -\frac{1}{2}\rangle \longrightarrow p \otimes n = \frac{1}{2} \times \frac{1}{2} = 1 + 3$

- The singlet would be $|pn\rangle = |0,0\rangle = \frac{1}{\sqrt{2}}\left(|\frac{1}{2}, \frac{1}{2}\rangle - |\frac{1}{2}, -\frac{1}{2}\rangle\right)$
- The triplet would be $|pp\rangle = |1,1\rangle$; $|pn\rangle = |1,0\rangle = \frac{1}{\sqrt{2}}\left(|\frac{1}{2}, \frac{1}{2}\rangle + |\frac{1}{2}, -\frac{1}{2}\rangle\right)$; $|nn\rangle = |1,-1\rangle$

If we do not see any pp or nn therefore we can conclude that pn is a singlet. We can test it through experiments like $p+p \longrightarrow d+\pi^+$; $p+n \longrightarrow d+\pi^0$; $n+n \longrightarrow d+\pi^-$

$|I, I_3\rangle_{in}$ $\qquad\qquad$ $|I, I_3\rangle_{fin}$

- $p + p \longrightarrow d + \pi^+$

(a) $|1,+1\rangle$ $\quad I_3 \quad \frac{1}{2} + \frac{1}{2} = 1 \qquad 0 + 1 = 1 \qquad |1,+1\rangle$

n.b. strong interactions conserve isospin

- $p + n \longrightarrow d + \pi^0$

(b) $\frac{1}{\sqrt{2}}(|1,0\rangle + |0,0\rangle)$ $\quad I_3 \quad \frac{1}{2} - \frac{1}{2} = 0 \qquad 0 + 0 = 0 \qquad |1,0\rangle$

- $n + n \longrightarrow d + \pi^-$

(c) $|1,-1\rangle$ $\quad I_3 \quad -\frac{1}{2} - \frac{1}{2} = -1 \qquad 0 - 1 = -1 \qquad |1,-1\rangle$

So the idea is to count the # of events and from them we obtain informations on σ and hence on the matrix elements. (n.b. # events $\propto \sigma$)

$$\left.\begin{array}{l} \sigma_a \propto |M_a|^2 \rho(p+p \longrightarrow d+\pi^+) \\ \sigma_b \propto |M_b|^2 \rho(p+n \longrightarrow d+\pi^0) \end{array}\right\} \longrightarrow R \equiv \frac{\sigma_a}{\sigma_b} = \frac{\#\, p+p \longrightarrow d+\pi^+}{\#\, p+n \longrightarrow d+\pi^0} = \frac{|M_a|^2}{|M_b|^2}\frac{\rho(d\pi^+)}{\rho(d\pi^0)}$$

Since $\frac{\Delta m_{p/n}}{m_p} \simeq 0.002$ and $\frac{\Delta m_\pi}{m_\pi} \simeq 0.036 \longrightarrow \rho(d\pi^+) \simeq \rho(d\pi^0) \longrightarrow R \simeq \frac{|M_a|^2}{|M_b|^2}$

$$M_a = \langle 1,1 | H_I | 1,1 \rangle$$

$$M_b = \frac{1}{\sqrt{2}} \left(\langle 1,0 | + \langle 0,0 | \right) H_I | 1,0 \rangle = \frac{1}{\sqrt{2}} \langle 1,0 | H_I | 1,0 \rangle + \frac{1}{\sqrt{2}} \langle 0,0 | H_I | 1,0 \rangle$$

If our hypothesis is true and isospin is conserved in strong interaction then we have that:

$$H_I | 1,0 \rangle = k \, | 1,0 \rangle \longrightarrow M_b = \frac{1}{\sqrt{2}} \langle 1,0 | H_I | 1,0 \rangle + k \frac{1}{\sqrt{2}} \underbrace{\cancel{\langle 0,0 | 1,0 \rangle}}_{=0}$$

$$\longrightarrow R = \frac{\sigma_a}{\sigma_b} \simeq \frac{1}{\left| \frac{1}{\sqrt{2}} \right|^2} \simeq 2 \quad \text{confirmed by} \quad \frac{\# \pi^+ d}{\# \pi^0 d}$$

and this is the experimental proof that isospin exists.

- The same thing has been repeated with beam of charged pions : $\pi^+ + p/n \longrightarrow \ldots$
  Below there are the possible processes :

$|\pi^+\rangle = |1,1\rangle \quad |p\rangle = |\tfrac{1}{2}, \tfrac{1}{2}\rangle$

$\pi^+ + p \longrightarrow \pi^+ + p$ (a)

$\cancel{\pi^+ + p \longrightarrow \pi^{++} + n}$ $\longrightarrow$ there's no π^{++} $\rightarrow$ not possible

$\cancel{\pi^+ + p \longrightarrow \pi^+ + \pi^+ + n}$ $\longrightarrow$ it's possible but we're not looking for 3 particle states (only 2)

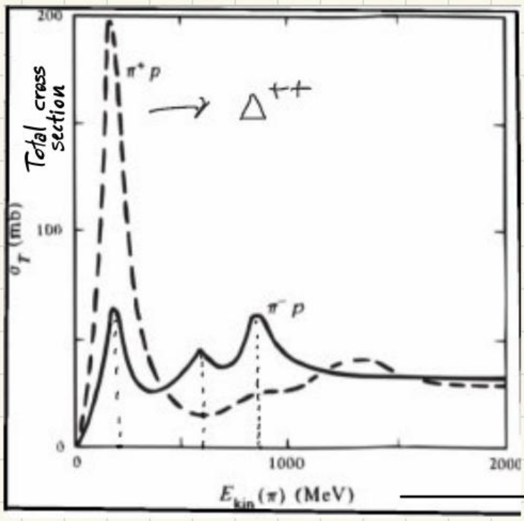
$\pi^+ + n \longrightarrow \pi^+ + n$ (d)

$\pi^+ + n \longrightarrow \pi^0 + p$ (g)

$\pi^- + p \longrightarrow \pi^- + p$ (c)

$\pi^- + p \longrightarrow \pi^0 + n$ (j)

With the same computations done above one finds: $\longrightarrow \dfrac{\sigma(\pi^+ + p)}{\sigma(\pi^- + p)} \simeq 3$



This is what we find doing experiments

$(E_\pi, \vec{p}_\pi)$ $\quad$ $(m_p, \vec{0})$

$\pi \bullet \longrightarrow \bullet\, p$

$\longrightarrow \dfrac{\sigma(\pi^+ + p)}{\sigma(\pi^- + p)} \simeq 3$ confirmed by data.

$\longrightarrow E^{\pi}_{kin} = E_\pi - m_\pi$

N.B. We have different resonances. The most stable is the one with the narrower pick (since the width of the pick goes like $\Gamma \sim \frac{1}{\tau}$). From the plot we could extract infos about the particles we see in the resonance. From the central point of the resonance we get the mass a from the width the lifetime of the particle. Then the most stable resonance comes from $\pi^+ p$ then its charge most be +2. Therefore we can already say that it is not a meson, since with a meson $q\bar{q}$ the best we can do is a charge $\frac{2}{3} + \frac{1}{3} = +1$. So we could say that maybe it's a uuu or there may be charm quark. Today we know this particle is a Δ^{++}, an hadronic resonance discovered back in the 50's.

G PARITY

We would like to add another conserved quantity of strong interactions: G

	EM.	STRONG	WEAK
$\mathbb{P}$	✓	✓	✗
$\mathbb{C}$	✓	✓	✗
I	✗	✓	✗
G	✗	✓	✗

G parity is a mixture of $\mathbb{C}$ parity and a rotation $\mathbb{R}_2$ in isospin space around I_2.

- $\mathbb{C}$ parity transforms the particle in its antiparticle $\quad \mathbb{C}\,|\psi\rangle = (-1)^{L+S}\,|\bar{\psi}\rangle$
- $\mathbb{R}_2$ is a rotation around I_2 in isospin space $\quad \mathbb{R}_2 = e^{i\pi I_2} \rightarrow \mathbb{R}_2\,|I, I_3\rangle = (-1)^{I-I_3}|I, -I_3\rangle$

$$\longrightarrow \boxed{G = \mathbb{C}\times\mathbb{R}_2 \qquad \text{with} \qquad G = (-1)^{L+S+I}}$$

N.B. It is true only for I = integers

We can show that it is conserved in strong interactions.

$$\left.\begin{aligned}
&\mathbb{C}\,|\pi^+\rangle = -\,|\pi^-\rangle \\
&\mathbb{C}\,|\pi^-\rangle = -\,|\pi^+\rangle \\
&\mathbb{C}\,|\pi^0\rangle = +\,|\pi^0\rangle \\
&\mathbb{R}_2\,|\pi^+\rangle = (-1)^{1-1}\,|\pi^-\rangle = |\pi^-\rangle \\
&\mathbb{R}_2\,|\pi^-\rangle = (-1)^{1-(-1)}\,|\pi^+\rangle = |\pi^+\rangle \\
&\mathbb{R}_2\,|\pi^0\rangle = (-1)^{1-0}\,|\pi^0\rangle = -\,|\pi^0\rangle
\end{aligned}\right\}
\begin{aligned}
&(\mathbb{C}+\mathbb{R}_2)\,|\pi^+\rangle = (-1)(+1)\,|\pi^+\rangle = (-1)\,|\pi^+\rangle \\
&(\mathbb{C}+\mathbb{R}_2)\,|\pi^-\rangle = (-1)(+1)\,|\pi^-\rangle = (-1)\,|\pi^-\rangle \\
&(\mathbb{C}+\mathbb{R}_2)\,|\pi^0\rangle = (+1)(-1)\,|\pi^0\rangle = (-1)\,|\pi^0\rangle
\end{aligned}$$

STRANGE PARTICLES

The strange particles are produced strongly but they can decay with long lifetime (weak interactions). Examples of strange particles are $K^\pm, K^0, \Lambda, B, D, \ldots$. We'll see that they're made by strange quark.

GELL MAN - NISHIJIMA FORMULA

At the time there was a huge list of particles, each own with its quantum numbers and the theorist tried to put some orders in all these numbers. The idea that these numbers are related came from Gell-Mann and Nishijima in 1956.
Looking at a nucleus with Z protons and $A-Z$ neutrons it has an isospin:

$$I_3 = Z\times\frac{1}{2} + (A-Z)\left(-\frac{1}{2}\right) = Z\left(\frac{1}{2}+\frac{1}{2}\right) - \frac{1}{2}A = Z - \frac{A}{2}$$

$$\longrightarrow Z = I_3 + \frac{A}{2}$$

Calling $Q \equiv Z$ and $B \equiv A$:

$$\boxed{Q = I_3 + \frac{B}{2}}$$

Gell-Man Nishijima formula

This formula worked for every baryon and for every meson (like $\pi^+ : +1 + \frac{0}{2} = +1$). The problem came with strange particles $K^+, K^-, K^0, \bar{K}^0$. To account for them a new quantum number was introduced: the strangeness S. Therefore the formula was updated into:

$$\boxed{Q = I_3 + \frac{B+S}{2}} \qquad \text{where} \quad \boxed{B+S = Y} \text{ Hyper-Charge}$$

What we know experimentally is that there were the pions that have similar masses then there were the Kaons also with similar masses, then after a certain gap an other set of particles.

Name	$\pi^\pm$	π^0	$K^\pm$	K^0	η	p	n	Λ	$\Sigma^{\pm,0}$	Δ
Mass (MeV)	140	135	494	498	548	938	940	1116	1190	1232
Charge	±1	0	±1	0	0	1	0	0	±1,0	2,±1,0
Parity	–	–	–	–	–	+	+	+	+	+
Baryon n.	0	0	0	0	0	1	1	1	1	1
Spin	0	0	0	0	0	½	½	½	½	3/2

The idea is to try to account for this patterns using symmetries, using the same approach of Heisemberg with the isospin idea.

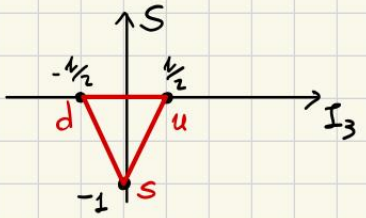
STATIC QUARK MODEL

The idea that there was a symmetry was called the Eightfold way (1961-1964). Gell-Man an Zweig in 1964 did something similar to what Heisemberg done using group theory. Their idea is that hadrons are multiplets of SU(3) therefore there are 3 quarks:

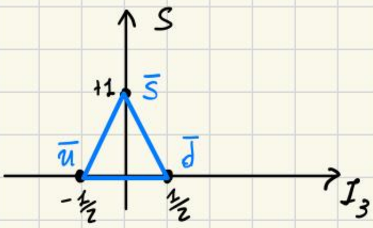
$$\begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

which are the fondamental representation of SU(3) which has $3^2 - 1 = 8$ generators where 2 of them are diagonal: I_3, Strangeness. This idea can help us to see how hadrons are produced by quarks. The 3 quarks have the following values of isospin and strangeness

	I_3	S
u	$+1/2$	0
d	$-1/2$	0
s	0	-1



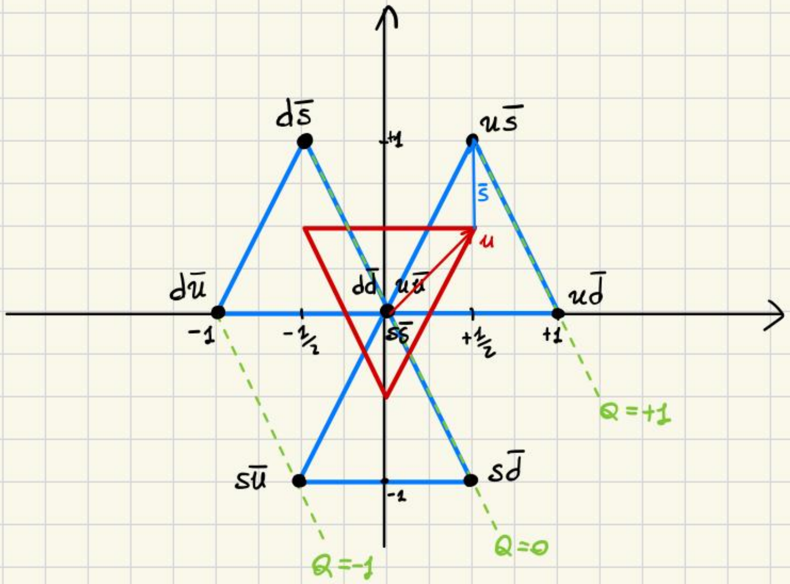
	I_3	S
$\bar{u}$	$-1/2$	0
$\bar{d}$	$+1/2$	0
$\bar{s}$	0	$+1$



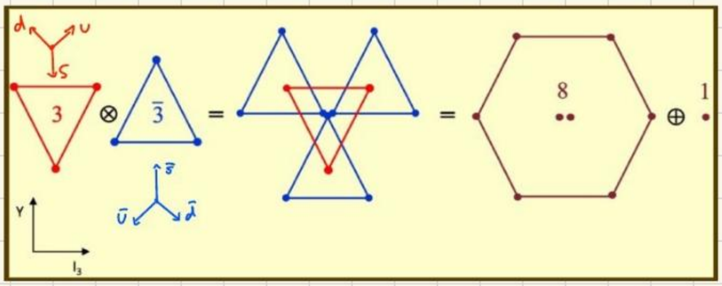
These 2 are our 2 lego blocks: we can combine them to obtain all the particles.

MESONS : $q_1 \bar{q}_2$

To obtain mesons (which are $3 \otimes \bar{3} = 9 = 8 \oplus 1$) we have to add a quark and an antiquark. We start from the origin of I_3, S plane then add a quark which is one of the arrow in the red triangle and then in the same way we add an antiquark (blue arrow). The state we end in is a meson. Graphically that is:



All the particles in the multiplets do not have the same mass and that is because the assumption that u, d and s have all the same mass is wrong (their masses have different values).
The fact that mesons are an octect and a singlet we picture in the following way:

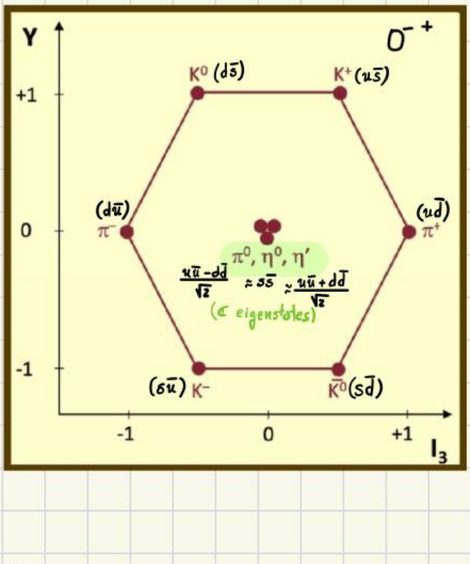


For the spin there are 2 possibilities:

$$q_i\bar{q}_j \begin{cases} S=0 \;:\; \Downarrow\Uparrow \\ S=1 \quad \Uparrow\Uparrow \end{cases}$$

we cannot discard one or the other from first principles so we have to account for both of them.

<u>Let's look at the case $S=0$ first</u>:



We are looking at particles which have $L=0$, that is because we are considering ground states (while excited states have $L>0$)
So we have for all these particles:

$$J = L \oplus S = 0$$

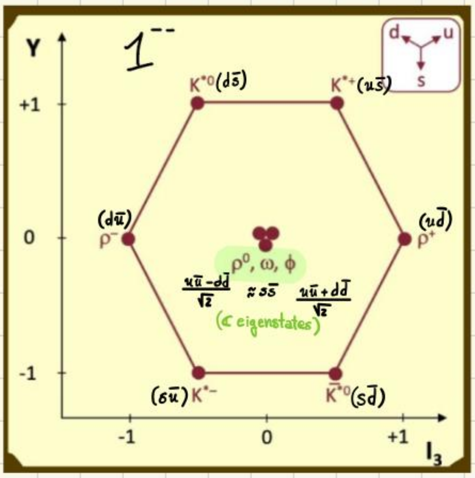
Therefore: $\begin{cases} P = (-1)^L P_i P_j = (-1)^0 (-1) = -1 \\ C = (-1)^{L+S} = (-1)^{0+0} = +1 \end{cases}$

So all the particles in this multiplet are identified by the quantum number:

$$\boxed{J^{PC} = 0^{-+}}$$

These particles are called pseudo-scalar mesons. (pseudoscalars because their parity is -1)

Let's now look at the case $s=1$:



In this case we have:

$$J = L \oplus S = 1$$

Therefore: $\begin{cases} P = (-1)^L \, P_i P_j = (-1)^0 (-1) = -1 \\ C = (-1)^{L+S} = (-1)^{0+1} = -1 \end{cases}$

$$\longrightarrow \quad \boxed{J^{PC} = 1^{--}}$$

These particles are called **vector mesons.**

Each particle in the vector multiplet has a corresponding particle in the pseudo-scalar multiplet: they have different names even though the flavor content is the same because particles in the vector multiplet are states of an higher energy. (i.e. particles with a greater mass) because J is higher.

About the 3 particles, the 3 purey theoretical states predicted by Group Theory are $u\bar{u}, d\bar{d}, s\bar{s}$. The 3 physical states that have actually been observed are instead π^0, η^0, η'. To obtain the physical states we have to pick combinations of the states we've found from the theory.

$$\left.\begin{aligned} \psi_{8,1} &= \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \qquad I=1 \\ \psi_{8,0} &= \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}) \qquad I=0 \end{aligned}\right\} \text{ Octect}$$

$$\psi_{1,0} = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}) \qquad I=0 \quad \Big\} \text{ Singlet}$$

The state $\psi_{8,1}$ has isospin 1 and it hasn't any $s\bar{s}$ content so it is the lightest of the 3, and so it is a candidate to be the π^0.
Of the other 2 states we take a combination and then we fit it to data to see what are the particles we have. We write these general 2 orthogonal combinations:

$$\bigstar \quad \boxed{\begin{aligned} f_i' &= \psi_{8,0}\cos\theta_i - \psi_{1,0}\sin\theta_i \\ f_i &= \psi_{8,0}\cos\theta_i + \psi_{1,0}\sin\theta_i \end{aligned}}$$

$i = ps, v$ (pseudoscalar, vector)
θ_{ps}: pseudo-scalar meson mixing angle
θ_v: vector meson mixing angle.

Then we fit them to data to see what are the particles we have. We can also rewrite

$$f_i' = \cos\theta_i \left[\psi_{8,0} - \psi_{1,0}\frac{\sin\theta_i}{\cos\theta_i}\right] = \cos\theta_i \left[\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}) - \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})\frac{\sin\theta_i}{\cos\theta_i}\right]$$

rewritten in this way we can also see that if the angle θ_i happens to be equal to the angle $\theta \simeq 35{,}5°$ at which $\sin\theta/\cos\theta = \frac{1}{\sqrt{2}}$ then $f' \propto s\bar{s}$. Diagonalizing this $\bigstar$ problem we find the masses as a function of the angle, then fitting the masses (experimentally measured) we infer the angle. Fitting to data one finds:

$$\left.\begin{aligned} \pi^0(140) &\approx \psi_{8,1}^{ps} = (u\bar{u} - d\bar{d})/\sqrt{2} \\ \eta(550) &= \psi_{8,0}^{ps}\cos\theta_{ps} - \psi_1^{ps}\sin\theta_{ps} \\ \eta'(960) &= \psi_{8,0}^{ps}\sin\theta_{ps} + \psi_1^{ps}\cos\theta_{ps} \end{aligned}\right\} \begin{aligned} J^P &= 0^-, \\ \theta_{ps} &\approx -25° \end{aligned}$$

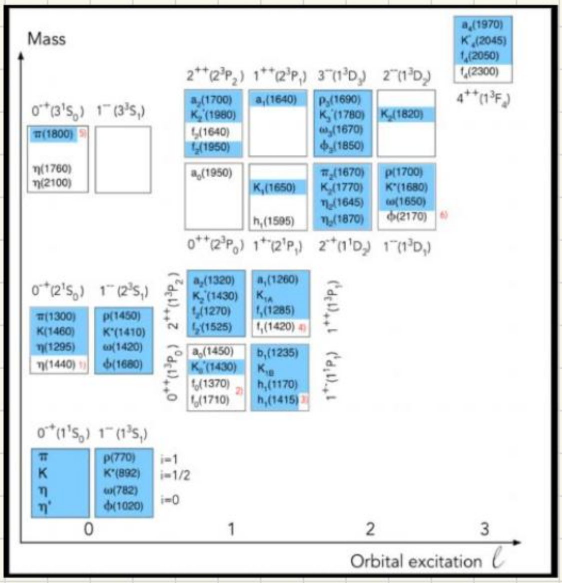
$$\left.\begin{aligned} \rho^0(770) &\approx \psi_{8,1}^{v} = (u\bar{u} - d\bar{d})/\sqrt{2} \\ \phi(1020) &= \psi_{8,0}^{v}\cos\theta_{v} - \psi_1^{v}\sin\theta_{v} \approx s\bar{s} \\ \omega(780) &= \psi_{8,0}^{v}\sin\theta_{v} + \psi_1^{v}\cos\theta_{v} \approx \\ &\approx (u\bar{u} + d\bar{d})/\sqrt{2} \end{aligned}\right\} \begin{aligned} J^P &= 1^-, \\ \theta_{vect} &\approx 36° \end{aligned}$$

The fit has been looking at the data available experimentally :

$n^{2s+1}\ell_J$	J^{PC}	I = 1 $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I = $\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $\bar{u}s$	I = 0 f'	I = 0 f
1^1S_0	0^{-+}	π	K	η	$\eta'(958)$
1^3S_1	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}[a]	$h_1(1415)$	$h_1(1170)$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}[a]	$f_1(1420)$	$f_1(1285)$
1^3P_2	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$[a]	$\eta_2(1870)$	$\eta_2(1645)$
1^3D_1	1^{--}	$\rho(1700)$	$K^*(1680)$[b]	$\phi(2170)$[d]	$\omega(1650)$
1^3D_2	2^{--}		$K_2(1820)$[a]		
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$
1^3F_4	4^{++}	$a_4(1970)$	$K_4^*(2045)$	$f_4(2300)$	$f_4(2050)$
1^3G_5	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$		
2^1S_0	0^{-+}	$\pi(1300)$	$K(1460)$	$\eta(1475)$[c]	$\eta(1295)$
2^3S_1	1^{--}	$\rho(1450)$	$K^*(1410)$[b]	$\phi(1680)$	$\omega(1420)$
2^3P_1	1^{++}	$a_1(1640)$			
2^3P_2	2^{++}	$a_2(1700)$	$K_2^*(1980)$	$f_2(1950)$	$f_2(1640)$

- Here on each column there is a different flavor content; going down there are excited states

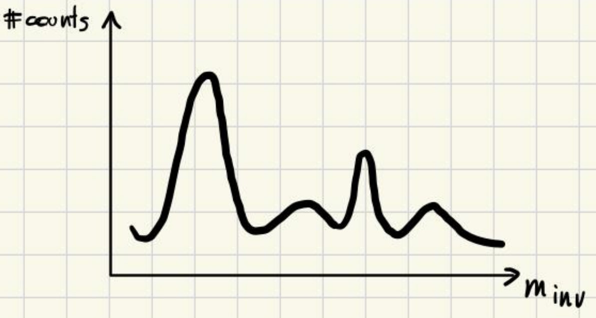
As we said, looking at the masses we can see that they have not the same value. This means that the symmetry we have is not truly exact : u, d and s have different masses. We can deduce looking to the flavors of the particles that the heavier ones are those with strange quarks. So the heaviest of them is the one with the flavor content $s\bar{s}$. We can look at the masses from the following plot:



- It is the same kind of plot we see while studying atomic physics but here the energy scale is huge. (~ MeV v.s. eV)
- Some particles have a width so large and so a lifetime very short that we see them just as intermediate states.
- There are about 20 different multiplets of particles.
- If the symmetry were exact the particles in the same multiplets should have the same mass, but since that's not the case this means that the SO(3) flavor symmetry is not exact, hence $m_u \neq m_d \neq m_s$ (from the exp. $m_d/m_u \simeq 2$, $m_s/m_d \simeq 20$).

How to produce such particles?

We can produce them in collisions, like: $p+p \longrightarrow \pi^+\pi^-\pi^0\pi^0 K^+K^- K^0\bar{K}^0 K^0\pi^+K^-$
In this process the baryon number is a problem, we have to produce at least 2 protons to balance the baryon number between initial and final state. It is much better to do $\pi^+ + p$: in this way we have to produce just 1 proton and this leaves more free energy to produce other things. The lighter particles ar the easier to produce because $\sigma \propto |M|^2 \rho(E)$: $|M|^2$ is pretty the same since we're considering strong interactions (since we have hadrons), the density of states (phase space) is larger if we have lighter particles in the initial state; this means that we can either produce more particles or produce particles that are heavier.



In an experiment we look at the counts we have as a function of the invariant mass of the system.

We can also produce such particles in decays. Here we have the constraint on the Qvalue that must be ≥ 0. The lighter is the mass in the final state (the larger is Q) the larger is the phase space.

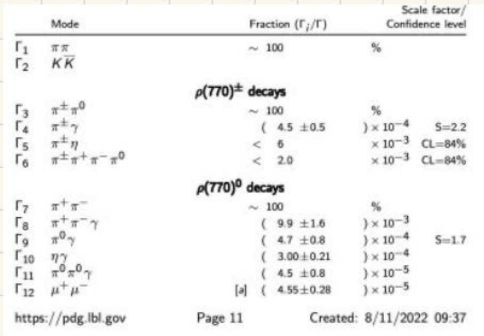
The important thing is that whatever is the method the process must conserves Q, B, L, S, I_3

Let's see how to produce $\rho^0(770)$

$\rho^0(770)$ is a vector meson (combination of u and $\bar{d}$). Intuitively the decay channels could be:

$$\rho^0 \to \pi^+\pi^-$$
$$\rho^0 \to \pi^0\pi^0$$
$$\rho^0 \to \cancel{K^+K^-}$$
$$\rho^0 \to \cancel{K^0K^0}$$

K is too heavy!
$m_\rho - m_K - m_K < 0 \to$ not allowed



	Mode		Fraction (Γ_i/Γ)		Scale factor/ Confidence level
Γ_1	$\pi\pi$		$\sim$ 100	%	
Γ_2	$K\bar{K}$				
			$\rho(770)^\pm$ decays		
Γ_3	$\pi^\pm\pi^0$		$\sim$ 100	%	
Γ_4	$\pi^\pm\gamma$		(4.5 $\pm$0.5	) $\times 10^{-4}$	S=2.2
Γ_5	$\pi^\pm\eta$		< 6	$\times 10^{-3}$	CL=84%
Γ_6	$\pi^\pm\pi^+\pi^-\pi^0$		< 2.0	$\times 10^{-3}$	CL=84%
			$\rho(770)^0$ decays		
Γ_7	$\pi^+\pi^-$		$\sim$ 100	%	
Γ_8	$\pi^+\pi^-\gamma$		(9.9 $\pm$1.6	) $\times 10^{-3}$	
Γ_9	$\pi^0\gamma$		(4.7 $\pm$0.8	) $\times 10^{-4}$	S=1.7
Γ_{10}	$\eta\gamma$		(3.00$\pm$0.21	) $\times 10^{-4}$	
Γ_{11}	$\pi^0\pi^0\gamma$		(4.5 $\pm$0.8	) $\times 10^{-5}$	
Γ_{12}	$\mu^+\mu^-$	[a]	(4.55$\pm$0.28	) $\times 10^{-5}$	

https://pdg.lbl.gov Page 11 Created: 8/11/2022 09:37

Seeing more in details we can see that there is not $\pi^0\pi^0$ in the possible decay channels. Why?
Or better, why $BF(\rho \to \pi^0\pi^0) = 0$?
This is due to several reasons:

1) <u>C parity</u>: $C_{\rho^0} = -1$ (p.d.g.) but $C_{\pi^0} = +1$: $\rho^0 \to \pi^0\pi^0$

C: -1 $\quad$ $+1$ $+1$

So, not possible in strong interactions (they must conserve C parity).

2) <u>Isospin</u>: we know that:

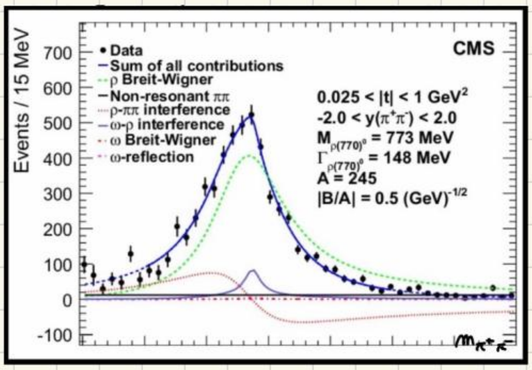
$$\begin{cases} |\rho^0\rangle = |I_F = 1; I_{3F} = 0\rangle \\ |\pi^0\rangle = |I = 1; I_3 = 0\rangle \end{cases}$$

$$\xrightarrow{\text{since}} \pi^0\pi^0 = 1 \otimes 1 = 0 \oplus 2 \longrightarrow |\pi^0\pi^0\rangle = \alpha|2,0\rangle + \beta|0,0\rangle$$

$$\longrightarrow \langle f|H_I|i\rangle = \langle 2,0|H_I|1,0\rangle + \langle 2,0|H_I|0,0\rangle = 0 \longrightarrow \langle\rho^0|\pi^0\pi^0\rangle = 0$$

3) <u>Spin Statistics</u>: ρ^0 is a boson, so we expect ψ_{ρ^0} to be symmetric. In the initial state we have a vector meson with 1 spin therefore $J=1$. If the final state were two π^0 this would mean that since $S_{\pi^0} = 0 \longrightarrow S_f = 0 \implies 1 \stackrel{!}{=} J_f = L_f + S_f \longrightarrow L_f = 1$ and this means that the wavefunction of the final state is not symmetric! (Absurd)

Looking at processes with $\pi^+\pi^-$ in the final state we observe:



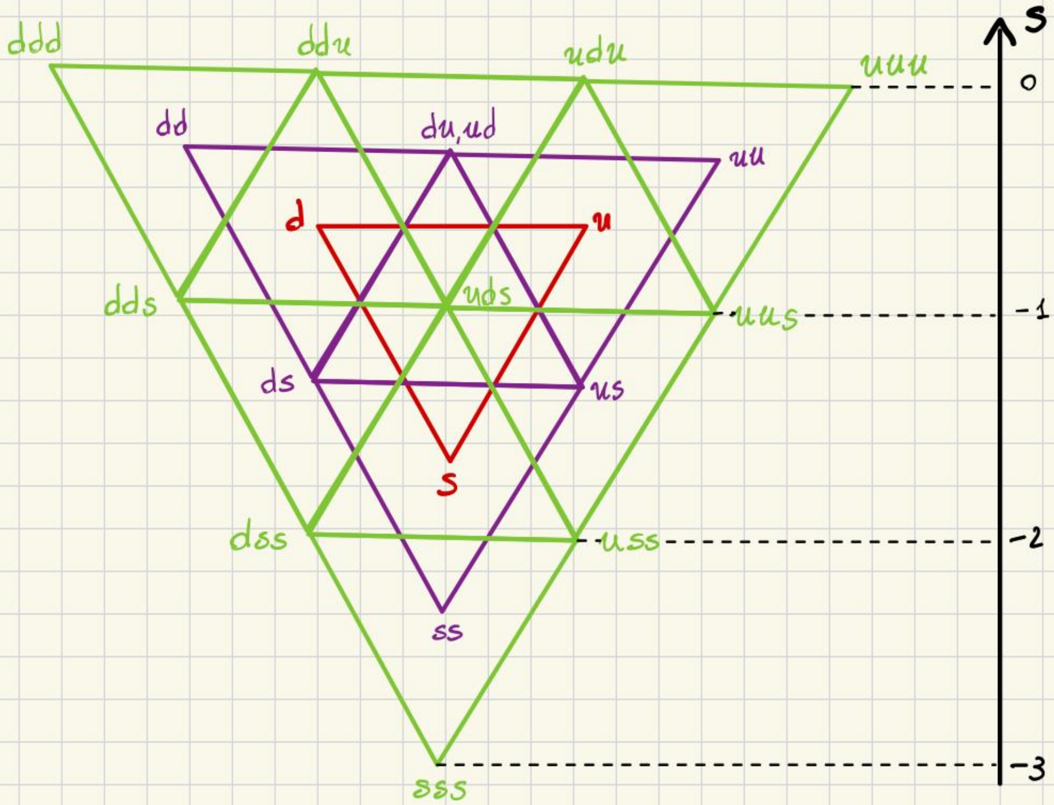
- What we see is the sum of all contributions.
- Among them there is also the interference between the ρ and other states with the same quark content.
- From the plot we see that $\Gamma_\rho \sim 150$ MeV, $m_\rho \sim 770$ MeV
- The background is subtracted and so it can happen that we have a number of events associated with the signal that is negative (obviously the signal in such cases should be compatible with 0 between the uncertainties, if that's not the case the BG has not been extimated.

BARYONS : $q_1 q_2 q_3$

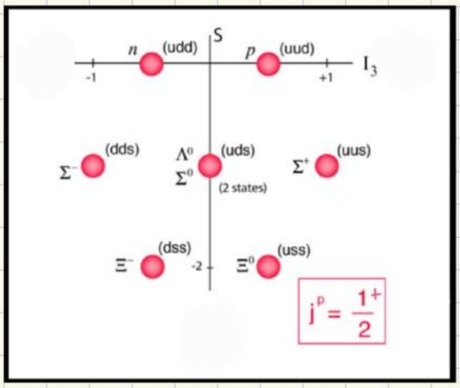
Looking at baryons we'll see that the predictions of Gell-Mann theory came into play. A baryon is made up of three quarks, so:

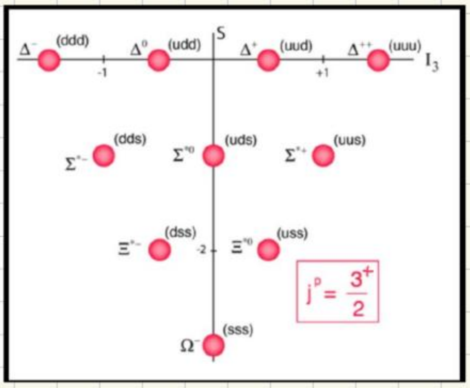
$$B = q_1 q_2 q_3 = 3 \otimes 3 \otimes 3 = \underset{\substack{\text{decoplet} \\ \text{(symmetric)}}}{10} \oplus \underset{\substack{\text{octect} \\ M_{23} \\ \text{(symm.} \\ \text{under } 2\leftrightarrow 3)}}{8} \oplus \underset{\substack{\text{octect} \\ M_{12} \\ \text{(symm.} \\ \text{under } 1\leftrightarrow 2)}}{8} \oplus \underset{\substack{\text{singlet} \\ \text{(antisymmetric)}}}{1} = 27 \text{ possible states}$$

We now have to add quarks as vectors as we've already seen with mesons and what we find is



With baryons we have 3 quarks, so an odd number of particles and so it has no sense to define a C-parity, so we just look at multiplets labelled by J^P.
Looking at ground states ($L=0$) we have:





$\uparrow\uparrow\downarrow$ (2nd combination) $\rightarrow S = \frac{1}{2}$

$\longrightarrow J = L + S = \frac{1}{2}$

$P = (-1)^L P_1 P_2 P_3 = (-1)^0 = +1$

$\rightarrow J^P = \frac{1}{2}^+$

$\uparrow\uparrow\uparrow$ (1st combination) $\rightarrow S = \frac{3}{2}$

$\longrightarrow J = L + S = \frac{3}{2}$

$P = (-1)^L P_1 P_2 P_3 = (-1)^0 = +1$

$\rightarrow J^P = \frac{3}{2}^+$

So, the states we're looking for are:

Baryons	qqq	J^P	I	I_3	S	$Q^{(1)}$	mass (MeV)
p, n	uud, udd	$½^+$	½	½, -½	0	1, 0	940
Λ	uds	$½^+$	0	0	-1	0	1115
$\Sigma^+, \Sigma^0, \Sigma^-$	uus, uds, dds	$½^+$	1	1, 0, -1	-1	1, 0, -1	1190
Ξ^0, Ξ^-	uss, dss	$½^+$	½	½,-½	-2	1, 0	1320
$\Delta^{++}, \Delta^+, \Delta^0, \Delta^-$	uuu, uud, udd, ddd	$3/2^+$	3/2	3/2, 1/2, -1/2, -3/2	0	2, 1, 0, -1	1230
$\Sigma^{*+}, \Sigma^{*0}, \Sigma^{*-}$	uus, uds, dds	$3/2^+$	1	1, 0, -1	-1	1, 0, -1	1385
Ξ^{*0}, Ξ^{*-}	uss, dss	$3/2^+$	½	½,-½	-2	1, 0	1530
Ω^-	sss	$3/2^+$	0	0	-3	-1	1670

~150 MeV
~150 MeV
~150 MeV

We expect the masses of the baryons to be similar if the quark content is the same, and that is indeed what we find. Still there are some corrections due to angular momentum structure. However respect to the meson's case here the masses are more similar. For a fixed strangeness the mass is the same (in that given row). There is a percentual mass variation when we increase strangenes: putting an s quark in place of u or d quark we obtain a mass variation $\Delta m \simeq 150$ MeV.

Gell-Mann theory for baryons reproduced data but it also made a prediction: in 1962 the $\Omega^-(sss)$ of the decuplet have not been seen yet. In the same year was observed the Ξ^* with a mass of $m \simeq 1530$ MeV. So were the theory to be correct one should expect the Ω^- to have a mass of $m \simeq 1530 + 150 = 1680$ MeV. Then from the Gell-Mann Nishijima formula we know that if we change isospin we also change the charge. Ω^- has an isospin $=0$, therefore:

$$I_3 = 0 \quad ; \quad B = \frac{1}{3} \times 3 = 1 \quad ; \quad S = (-1) \times 3 = -3 \quad \longrightarrow \quad Q = I_3 + \frac{B+S}{2} = -1$$

The predicted lifetime was $\tau \sim 10^{-10}$ s (weak decay)

Suppose we produce a Ω^- with a mass $m \simeq 1680$ MeV and we ask ourself for the possible decay modes.

- From STRONG decay we have $\Delta S = 0$, so we expect something like:

$$\begin{array}{cccc} & \Omega^- & \longrightarrow & X + Y \\ B & 1 & & 1 \quad 0 \text{ (meson} \\ S & \cancel{-3} & & \cancel{-2} \quad \cancel{-1} \\ & \cancel{-3} & & \cancel{-1} \quad \cancel{-2} \\ & \cancel{-3} & & \cancel{-3} \quad \cancel{0} \\ & \cancel{-3} & & \cancel{0} \quad \cancel{-3} \end{array}$$

cannot happen because there is no other $S = -3$ particle.

$$\begin{array}{lcccc} & \Omega^- & \longrightarrow & \Xi^{*-} & + \overline{K}^0 \\ S & -3 & & -2 & -1 \\ Q & -1 & & -1 & 0 \\ B & 1 & & 1 & 0 \\ m\,(\text{MeV}) & 1680 & & 1530 & 500 \end{array} \longrightarrow Q_{value} < 0$$

- From EM decay we also have $\Delta S = 0$, and we expect something like:

cannot happen because there is no other $S = -3$ particle.

$$\begin{array}{lcccc} & \Omega^- & \longrightarrow & X^- + & Y + \gamma \\ S & -3 & & -3 & 0 \quad 0 \end{array}$$

- So we can have by exclusion only weak-decays with $\Delta S = 1$ like:

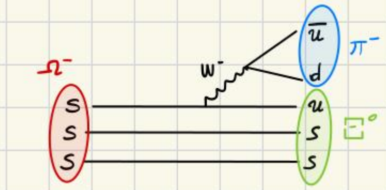
$$\begin{array}{lcccc} & \Omega^- & \longrightarrow & X + & Y \\ S & -3 & & -2 & 0 \\ & -3 & & -1 & -1 \end{array}$$

Two possible decays are:

$$\Omega^- \longrightarrow \Xi^0 + \pi^-$$
$$\qquad\quad \Xi^- + \pi^0$$

S: -3 $\quad -2 \quad 0 \longrightarrow \Delta S = 1$

m 1680 $\quad 1326 \quad 140 \longrightarrow Q > 0$



spectator diagram

$BF(\Omega^- \longrightarrow \Xi^0\pi^-) = 24\%$

Internal diagram

$BF(\Omega^- \longrightarrow \Xi^-\pi^0) = 9\%$

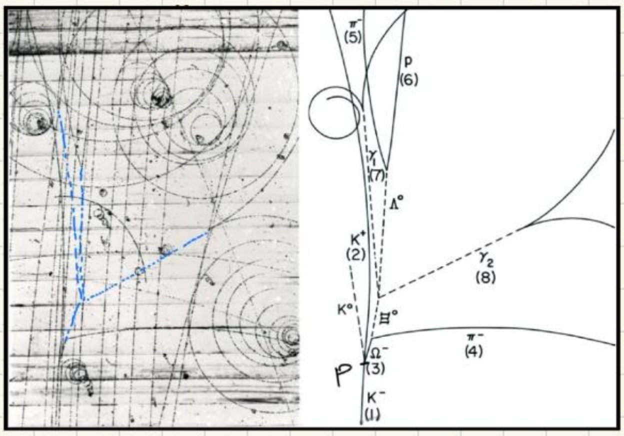
$$\Omega^- \longrightarrow \Lambda^0 + K^-$$

S: -3 $\quad -1 \quad -1 \longrightarrow \Delta S = 1$

m: 1680 $\quad 1115 \quad 500 \longrightarrow Q > 0$

$BF(\Omega^- \longrightarrow \Lambda^0 K^-) = 68\%$

The decay $\Omega^- \longrightarrow \Xi^0\pi^-$ was observed in 1964 at Brookhaven National Laboratories by Nick Samios using bubble chambers:



$p + p \longrightarrow p + p + K^- + K^+ \longrightarrow K^+$ beam

$K^+ + p \longrightarrow \Omega^- + K^+ + K^0$

$\quad \hookrightarrow \Xi^0 + \pi^-$

$\qquad \hookrightarrow \Lambda^0 + \pi^0$

$\qquad\quad \pi^0 \hookrightarrow \gamma\gamma$

$\qquad\qquad \gamma \hookrightarrow e^+e^-$

$\qquad\qquad \gamma \hookrightarrow e^+e^-$

$\qquad\quad \Lambda^0 \hookrightarrow p\,\pi^-$

This was the first success of the Gell-Mann theory: the flavor model of SU(3) works well.

COLOR

Let's now study a Symmetry Problem that we have, it is small but relevant. If we consider the Δ^{++} baryon, it is given by $u^\uparrow u^\uparrow u^\uparrow$, so it has $L=0$, $s=\frac{3}{2} \longrightarrow J=\frac{3}{2}$, $P=(-1)^L=+1$. So the wave function of Δ^{++} is:

$$\psi_{\Delta^{++}} = \psi_{space}\,\psi_{spin}\,\psi_{flavor}$$

- ψ_{space}: symmetric since $L=0$
- ψ_{spin}: symmetric since all the 3 quarks have spin up
- ψ_{flavor}: symmetric since there are all up quarks $\longrightarrow$ symmetric under the exchange of $1\leftrightarrow2$, $2\leftrightarrow3$, $1\leftrightarrow3$.

The point is that Δ^{++} is a fermion! Therefore its wavefunction must be anti-symmetric. To solve this problem we have to add another piece: another quantum number, **COLOR**. Now the total wavefunction is:

$$\psi_{\Delta^{++}} = \underset{S}{\psi_{space}}\;\underset{S}{\psi_{spin}}\;\underset{S}{\psi_{flavor}}\;\underset{A}{\psi_{color}}$$

where ψ_{color} must be antisymmetric. Let's try to understand how many colors we need to solve the problem.

- 1 COLOR HYPOTHESIS (B) $\quad u^{\uparrow}_{B}\ u^{\uparrow}_{B}\ u^{\uparrow}_{B}$ still symmetric
- 2 COLOR HYPOTHESIS (R, B) $\quad u^{\uparrow}_{B}\ u^{\uparrow}_{R}\ u^{\uparrow}_{B}$ still symmetric under $1 \longleftrightarrow 3$
- 3 COLOR HYPOTHESIS (R G B) $\quad u^{\uparrow}_{R}\ u^{\uparrow}_{G}\ u^{\uparrow}_{B}$ antisymmetric!

So we need at least 3 colors, say Red, Blue, Green (R, G, B). For the Δ^{++} we can now write the part of the wavefunction related to color as:

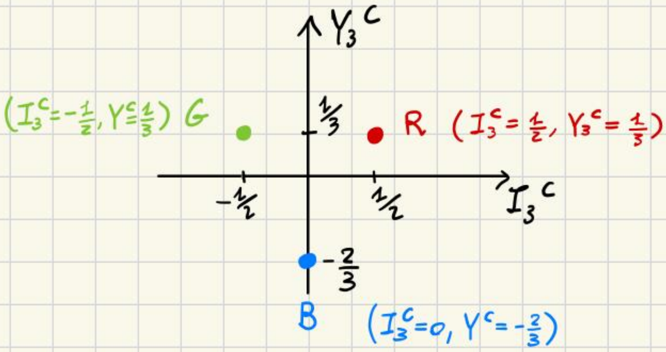
$$\psi_{color} = \frac{1}{\sqrt{6}} \left(|RGB\rangle + |BGR\rangle + |GRB\rangle - |BRG\rangle - |RGB\rangle - |GBR\rangle \right)$$

this is completely antisymmetric under the exchange of any couple of the 3 colors.

From a Group Theory point of view we're introducing an $SU(3)_{color}$ symmetry. $SU(3)_c$ has 8 generators where 2 of them are diagonal which for similarity with the flavor symmetry, that we've already studied, we call them Y^c, the color hypercharge, and I_3^c, the color Isospin. Defining the fundamental representation as:

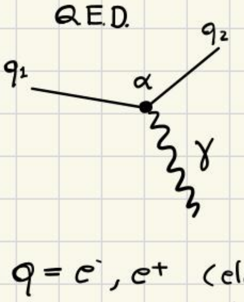
$$R = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad G = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

The fundamental lego brick is:

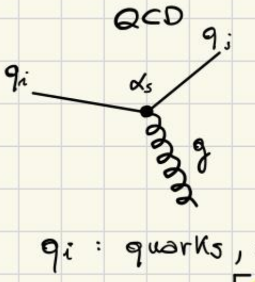


N.B. now adding all colors is possible to get something colorless.

The idea is that now we have a color charge as a property of quarks. Like electric charge in QED it determines how quarks (and gluons) interact through the strong force. This theory is called Quantum Chromo Dynamics (QCD).



$$\alpha = \frac{e^2}{4\pi}$$



$$\alpha_s = \frac{g^2}{4\pi}$$

$q = e^-, e^+$ (electron, positron)

γ: photon
- 1 type
- vector boson $S = +1$
- massless
- No electric charge $Q = 0$

q_i : quarks, antiquarks

g : gluons
- 8 types
- vector boson $s = 1$
- massless
- they carry color and anticolor charge since have to talk both with quarks and antiquarks

Moreover at that time there was also another big hypothesis / conjecture, called Color Confinement : in nature all physical particles are colorless i.e. they're color singlet.

So now we have to prove 3 things:

- Do quark exist ? (u, d, s)
- Do color exist? (Are they only 3?)
- Are there only colorless particles in nature?

we have to build observables to actually prove these 3 conjectures. N.B. The fact that every baryon and meson agree with the theory it's not a proof, it's just the theory filling the dots we already have. To actually prove the theory we need to measure a prediction of the theory itself. (otherwise it would remain only a conjecture). Another point we have to explain is why we see only 18 baryons and not 27 : this problem as we'll see is solved by color.

Baryon Wave function

$$B = q_1 q_2 q_3 \qquad \psi = \psi_{space}\,\psi_{spin}\,\psi_{flavor}\,\psi_{color}$$

- The space part is symmetric because we're looking at ground state $L=0$
- For the spin part we can have $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{2}+\frac{3}{2}$. Now we assume $s=\frac{3}{2}$ (symmetric) $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \left(\frac{3}{2}\right)_S + \left(\frac{1}{2}\right)_{M_{1,2}} + \left(\frac{1}{2}\right)_{M_{1,3}}$ $\uparrow\uparrow\uparrow$
- For the flavor we have that : $3_F \times 3_F \times 3_F = 10_{F,S} + 8_{F,M_{12}} + 8_{F,M_{23}} + 1_{F,A}$
- For the color we have that : $3_C \times 3_C \times 3_C = 10_{C,S} + 8_{C,M_{12}} + 8_{C,M_{23}} + 1_{C,A}$

If the conjecture of color confinement is true then we have to consider only the color singlet $1_{C,A}$ that is the antisymmetric combination given by :

$$1_{C,A} : \quad \psi_{color} = \frac{1}{\sqrt{6}}\left[|RGB\rangle + |GBR\rangle + |BRG\rangle - |GRB\rangle - |RBG\rangle - |BGR\rangle\right]$$

So we have :

$$\psi = \psi^{S}_{space} \times \psi^{3/2\,S}_{spin} \times \psi_{flavor} \times \psi^{A}_{color}$$

and hence here we can see that ψ_{flavor} has to be symmetric and therefore we have to choose the decuplet in flavor. In this way the total wavefunction is correctly antisymmetric :

$$\longrightarrow \boxed{\psi_{10} \equiv \psi^{S}_{space}\,\psi^{3/2\,S}_{spin}\,\psi^{S}_{\underset{10}{flavor}}\,\psi^{A}_{color}} \qquad \text{Baryon decuplet} \longleftrightarrow 10 \text{ physical states}$$

A flavor singlet would not be possible because (under the assumption of $L=0$, $S=3/2$, $1_{C,A}$) :

$$\longrightarrow \psi_1 \equiv \psi^{S}_{space}\,\psi^{3/2\,S}_{spin}\,\psi^{A}_{\underset{1}{flavor}}\,\psi^{A}_{color}$$

would be symmetric and this would violate Pauli exclusion principle. (However the flavor singlet could exist only in the case of spin $s=\frac{1}{2}$)

Let's now try to study the Baryon Octect. As we know the spatial part is symmetric since $L=0$, the color part is supposed to be the antisymmetric singlet. Therefore the product of the spin and flavor parts must be symmetric. In order to consider the octect we must take into account the total spin $1/2$ case (instead of $3/2$) :

$$\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \left(\frac{3}{2}\right)_S,\ \left(\frac{1}{2}\right)_{M_{1,2}},\ \left(\frac{1}{2}\right)_{M_{2,3}},\ \left(\frac{1}{2}\right)_{M_{1,3}}$$

Infact in that case if

$$\left(\frac{1}{2}\right)_{M_{1,2}} : \uparrow\uparrow\downarrow \xleftrightarrow{1-2} \uparrow\uparrow\downarrow$$

$$\left(\frac{1}{2}\right)_{M_{2,3}} : \uparrow\downarrow\downarrow \xleftrightarrow{2-3} \uparrow\downarrow\downarrow$$

$$\left(\frac{1}{2}\right)_{M_{1,3}} : \uparrow\downarrow\uparrow \xleftrightarrow{1-3} \uparrow\downarrow\uparrow$$

Then there is also $M_{1,3}$ that is just a linear combination of the other 2 $M_{1,3} = M_{12} + M_{23}$ (we obtain it from Clebsch Gordon coefficients.) In this case we have to symmetrize the product of the spin and flavor part as follows:

$$\boxed{\psi_{spin}\,\psi_{flavor} = \psi_{spin}^{S=\frac{1}{2},M_{12}} \cdot \psi_{flavor}^{M_{1,2}} + \psi_{spin}^{S=\frac{1}{2},M_{23}} \cdot \psi_{flavor}^{M_{2,3}} + \psi_{spin}^{S=\frac{1}{2},M_{13}} \psi_{flavor}^{M_{13}}}$$

Baryon octect $\leftrightarrow$ 8 physical states

Because of the relationship due to Clebsh-Gordon coefficients there are only 8 independent physical states. So this explains the particles we see in the Baryon octect. In total we therefore have 18 physical states (10 from decuplet with $S=\frac{3}{2}$ and 8 from octect with $S=\frac{1}{2}$).

Meson wave function

$$M = q_1 \bar{q}_2 \,, \qquad \psi_{meson} = \psi_{space}\,\psi_{spin}\,\psi_{flavor}\,\psi_{color}$$

- The space part is symmetric due to $L=0$
- The color part we have $3_c \times \bar{3}_c = 8_c + 1_c$ and we take the antisymmetric singlet due to color confinement:

$$\boxed{1_c : \frac{1}{\sqrt{3}}\left(|R\bar{R}\rangle + |B\bar{B}\rangle + |G\bar{G}\rangle\right)}$$

n.b. $\begin{cases} R+\bar{R}=0 \\ B+\bar{B}=0 \\ G+\bar{G}=0 \\ R+B+G=0 \end{cases}$

$$8_c : |R\bar{B}\rangle, |R\bar{G}\rangle, |B\bar{R}\rangle, |B\bar{G}\rangle, |G\bar{R}\rangle, |G\bar{B}\rangle, \frac{1}{\sqrt{2}}\left(|R\bar{R}\rangle - |B\bar{B}\rangle\right), \frac{1}{\sqrt{6}}\left(|R\bar{R}\rangle + |G\bar{G}\rangle - |2B\bar{B}\rangle\right)$$

- For the spin part we can have $\frac{1}{2} \times \frac{1}{2} = 0 + 1$ $\begin{cases} 0 : \text{pseudoscalar meson} \\ 1 : \text{vector mesons} \end{cases}$
- For the flavor part we have $3_F \times \bar{3}_F = 8_F + 1_F$

Since now we have a quark and an antiquark there's no Pauli exclusion principle to be respected and there's no need for the total wavefonction to be symmetric or antisymmetric

Therefore we understand that with just mesons there would not have been the need for color: color with mesons is not needed to explain anything. The idea of colors came up only to explain the Ω^-, Λ^-, Δ^{++} that would have otherwise a completely symmetric wavefunction.

GLUONS

The hypothesis are that gluons are:

- VECTOR BOSONS $S=1$
- MASSLESS
- 8 TYPES
- COLORED (or ANTICOLORED)

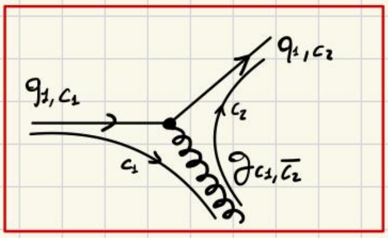
How do we know that they are 8? With an interaction between 2 quarks we have:
$3_c \times \bar{3}_c = 1_c + 8_c$:

- Octect: $|R\bar{B}\rangle, |R\bar{G}\rangle, |B\bar{R}\rangle, |B\bar{G}\rangle, |G\bar{R}\rangle, |G\bar{B}\rangle, \frac{1}{\sqrt{2}}(R\bar{R} - B\bar{B}), \frac{1}{\sqrt{6}}(R\bar{R} + G\bar{G} - 2B\bar{B})$
- Singlet: $\frac{1}{\sqrt{3}}\left(|R\bar{R}\rangle + |B\bar{B}\rangle + |G\bar{G}\rangle\right)$

In total we have 9 combinations, hence we expect 9 gluons. However a gluon that is a color singlet could be observed freely in nature, it would be an electrical neutral, spin 1, massless particle that we would be able to see in our detectors. This would imply long range strong interactions. In fact with a photon the propagator in mom. space is $-\frac{g^{\mu\nu}}{q^2}$ and performing a Fourier transform we find the potential in coordinate space to go like $1/r$.; with a gluon constructed exactly as the photon (so color neutral) we have an analogous result. Therefore we exclude the possibility to have a gluon in a color singlet (so colorless).

So we conclude that there is not colorless gluon but 8 colored gluons and those gluons and quarks do not exist free.

In general the vertex is:



i) gluons do not change flavor of quarks.

ii) color must be conserved

PROPERTIES OF COLORS

Colors must be conserved in all interactions. If we smash particles there is no color in initial state so also in the final state we have a colorless quantity.

$$\begin{array}{lllll} \bullet\ p + p & \longrightarrow & p + p & + & p + \bar{p} \\ & & p + p & + & \pi^+ + \pi^- \\ & & p + p & + & K^+ + K^- \\ C\ \ 0\ \ 0 & & 0\ \ \ 0 & & 0\ \ \ 0 \end{array}$$

$$\begin{array}{lll} \bullet\ e^+ + e^- & \longrightarrow & \pi^+ + \pi^- \\ C\ \ 0\ \ \ 0 & & 0\ \ \ \ 0 \end{array}$$

If we create a flavor with a certain color then we also create an antiflavor with the related anticolor like:

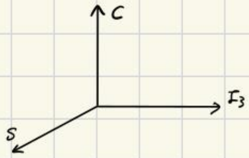
$$\begin{array}{lll} \bullet\ e^+ e^- & \longrightarrow & u_R\, \bar{u}_R \\ C\ \ 0 + 0 & & R + \bar{R} = 0 \end{array}$$

However we'll never see something like this because there are no free quarks in nature, we have to first dress them as hadrons. like: $e^+e^- \longrightarrow \pi^+(u_R \bar{d}_{\bar{R}}) + \pi^-(\bar{u}_{\bar{R}} d_R)$

EXTENSION TO MORE QUARKS

So far we've seen Isospin with SU(2) that is the symmetry between n and p. Taking also strangeness into account we find the quark model with SU(3) that is the symmetry between the the baryons and between the mesons. In general with SU(N) we have N^2-1 generators, of which $N-1$ are diagonal and hence corresponding to physical observables: for SU(2) we have I_3 (with $m_p \simeq m_n$) and for SU(3) I_3, S (with $m_u \simeq m_d \simeq m_s$). The mass difference tells us that the flavor symmetry is not exact).

If we consider also a 4th quark, the charm c, then we have to consider the symmetry of flavor $SU(4)_F$. In $SU(4)_F$ there are $N^2-1=15$ generators where $N-1=3$ of which are diagonal I_3, S, C (3 physical observables) where C is the CHARM:



the fundametal representation is

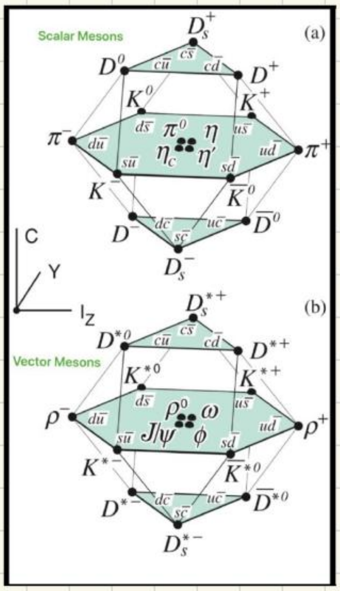
$$u = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \quad d = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \quad s = \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \quad c = \begin{pmatrix}0\\0\\0\\1\end{pmatrix} \quad \longleftrightarrow \quad \begin{pmatrix}u\\d\\s\\c\end{pmatrix}$$

With this symmetry the Gell-Mann Nishijima formula becomes:

$$\boxed{Q = I_3 + \frac{B+S+C}{2}}$$

MESONS

Mesons are now described by $4_F \times \bar{4}_F = 1_F + 15_F$. So, before we visualized the 3×3 as an octec plus a singlet, analogously we can now visualize 4×4 making use of $N-1 = 3$ dimensions:



- In the plane $C=0$ there are the same mesons we've encountered before.
- But now in the plane $C\neq0$ we have new mesons like $c\bar{u} = D^0$; $c\bar{d} \equiv D^+$; $c\bar{s} \equiv D_s^+$ (the heaviest because of s)
- We have $m_{D^0} \simeq 1860$ MeV and $m_{D^*} \simeq 2010$ MeV
- For the singlet we have a combination of $c\bar{c}, u\bar{u}, d\bar{d}$ and $s\bar{s}$ and experimentally we have to see what are the mixing angles between the families.

If we also add another quark, the beauty b, we go to an higher number of dimensions: $SU(5)_F$. In that case we have $N-1 = 4$ diagonal generators I_3, S, C, b. where b is the BEAUTY. The Gell-Mann Nishijima formula becomes:

$$\boxed{Q = I_3 + \frac{B+S+C+b}{2}}$$

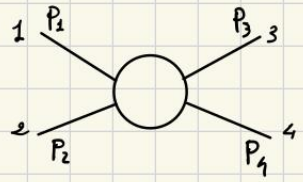
MESONS

Mesons are now described by $5\times\bar{5} = 1 + 24$. We can visualize just 3D slices of the whole construction. We find new mesons like: $B^-: b\bar{u}$ or $\bar{B}^0: b\bar{d}$. The mass of the B mesons is estimated to be $m_B \simeq 5279$ MeV (measured by Shahram Rahatlou).
It's interesting to note that the name mesons was used because the first observed mesons had a mass between the mass of electron and the mass of protons/neutrons, now it's no more the case!
The higher is the mass of the particle the more decay modes there are (because we have a larger phase space

- for the ρ^0 we have just: $\rho^0 \longrightarrow \pi^+\pi^-$ ($m_{\rho^0} \simeq 770$ MeV)
- for π^0 we have just 3 hadronic modes
- now with B^0 we can have a lot of decay modes (approx 520)

USING QED TO DISCOVER THAT QUARKS AND COLORS EXIST

The process we want to study is the 2 body scattering



we already saw the differential cross section in the c.o.m. frame:

$$\frac{d\sigma}{d\Omega} \propto \frac{1}{(E_1+E_2)} \frac{|\vec{p}_{out}|}{|\vec{p}_{in}|} |M|^2$$

Since we want to deal with quantities which are *Lorentz invariant* we introduce the Mandelstan variables s, t, u. Due to energy conservation we have that $p_1 + p_2 = p_3 + p_4$ and this tells us that there are just 2 independent variables and all the others are built out of them. The 3 Mandelstan variables are:

$$\boxed{\begin{aligned} s &= (p_1+p_2)^2 = p_1^2 + p_2^2 + 2p_1\cdot p_2 = m_1^2 + m_2^2 + 2p_1\cdot p_2 \\ t &= (p_1-p_3)^2 = m_3^2 + p_1^2 - 2p_1\cdot p_3 \\ s &= (p_1-p_4)^2 = m_4^2 + p_1^2 - 2p_1\cdot p_4 \end{aligned}}$$

These are the square of 4 vectors, they are invariant by construction, so we can compute them in any frame of reference. *If we add them we find:*

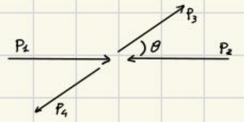
$$s+t+u = m_1^2 + m_2^2 + m_3^2 + m_4^2 + 2p_1\cdot p_2 + 2p_1\cdot p_1 - 2p_1\cdot p_3 - 2p_1\cdot p_4 =$$

$$= \sum_i m_i^2 + 2p_1\cdot(p_2+p_1-p_3-p_4) \overset{(p_1+p_2=p_3+p_4)}{=} \sum_i m_i^2$$

$$\longrightarrow \boxed{s+t+u = \sum_i m_i^2}$$

Note that at high energy limit $E_i \gg m_i \longrightarrow s+t+u \simeq 0$

Let's see an explicit computation in the high energy limit:

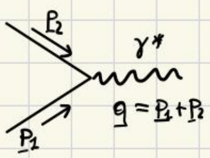


$$p_1 = (E_1, p, 0, 0) \qquad p_3 = (E_3, p_{out}\cos\theta, p_{out}\sin\theta, 0)$$
$$p_2 = (E_2, -p, 0, 0) \qquad p_4 = (E_4, -p_{out}\cos\theta, -p_{out}\sin\theta, 0)$$

N.B. There is no φ dependence because since the potential goes like $1/r$ there is angular momentum conservation and so the result is completely symmetric around φ, so we can just look at $\varphi = 0$.

$$s = (p_1+p_2)^2 = (E_1+E_2)^2 = (2E)^2 = 4E^2 \text{ (we choose } E_1 = E_2 \text{, symm. beams)}$$

Now we can understand why the so called *annihilation channel* is called s-channel.

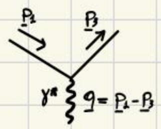


The mediator has a 4-momentum $q = p_1 + p_2 \longrightarrow q^2 = s$! This implies:

$$-\frac{g_{\mu\nu}}{q^2} \sim \frac{1}{s}$$

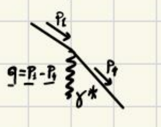
Regarding the others variables we have:

$$t = (p_1-p_3)^2 = (p_2-p_4)^2 = \dots \simeq -\frac{1}{2}s\cdot(1-\cos\theta) \longrightarrow$$



$$\longrightarrow -\frac{g_{\mu\nu}}{q^2} \sim \frac{1}{t}$$

$$u = (p_1-p_4)^2 = (p_2-p_3)^2 = \dots \simeq -\frac{1}{2}s\cdot(1+\cos\theta) \longrightarrow$$

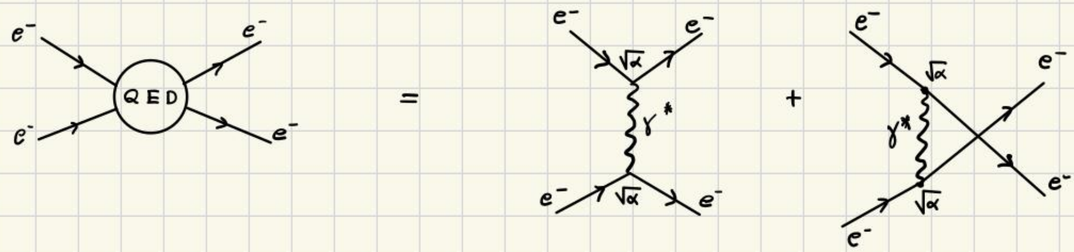


$$\longrightarrow -\frac{g_{\mu\nu}}{q^2} \sim \frac{1}{u}$$

In particular let's consider a simple QED process, an elastic scattering, the Möller Scattering:

$$e^-e^- \longrightarrow e^-e^-$$

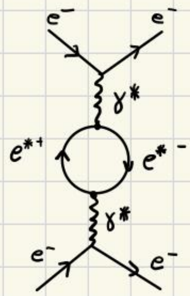
t-channel u-channel



Theoretically we add 2 diagrams together. Experimentally we send 2 electrons in and we see 2 electrons coming out, we do not know what happens. N.B. Feynman diagrams are just tools to make calculations. We have a $\sqrt{\alpha}$ fore each vertex so for the 2 above diagrams we have:

$$\mathcal{M}_1 \propto \alpha_{EM} \dots \quad ; \quad \mathcal{M}_2 \propto \alpha_{EM}, \dots$$

Then we could also add other diagrams at higher orders like:



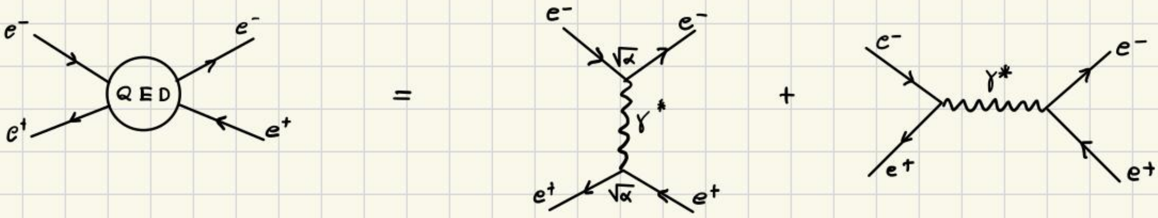
In this case $\mathcal{M}_3 \propto \alpha_{EM} \cdot \alpha_{EM} = \alpha_{EM}^2$ and since $\alpha = \frac{1}{137}$ this diagram is $\sim 10^2$ smaller than the previous ones, so we can just forget about it at 1% of accuracy.

This process is simple but it's not so useful because we want to create new particles.

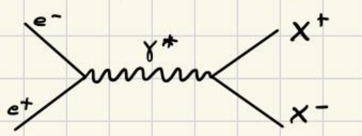
So let's consider the Bhabha scattering

$$e^-e^+ \longrightarrow e^-e^+$$

t-channel s-channel (Annihilation)



Experimentally we cannot tell which of the 2 happens, we just see $e^- + e^+$ coming in and $e^- + e^+$ coming out. N.B. with Möller scattering we cannot have an s channel due to charge conservation. The s-channel is the most useful because thanks to annihilation, as long as we conserve energy and the quantum numbers, we can produce what we want. (there can also be intermediate states):



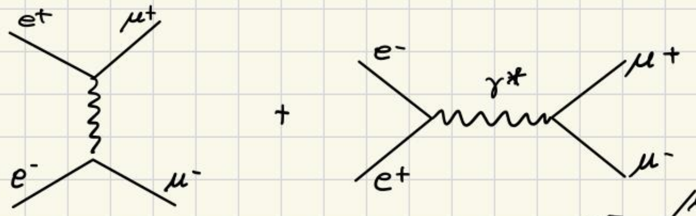
As a curiosity note that Bhabha scattering is a so well known process and calculated with so high precision that is used to calibrate the luminosity of the apparatus (it is a luminosity counter):

$$N = \sigma \cdot \mathcal{L}_{inst} \cdot \Delta t \longrightarrow N(\text{Bhabha scatt.}) = N(e^-e^+ \to e^-e^+) = \sigma_{Bhabha} \cdot \mathcal{L}_{inst} \cdot \Delta t$$

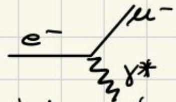
So by counting N in a time Δt and knowing σ^{th}_{Bhabha} we can obtain $\mathcal{L}_{inst}$.

The same thing can be done with μ in the final state $e^+e^- \longrightarrow \mu^+\mu^-$. A priori one could say that also here we have

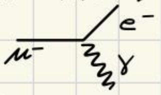
$e^+e^- \longrightarrow \mu^+\mu^-$ =



however the 1st diagram is not possible because if

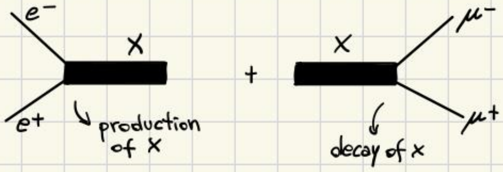


is possible than also



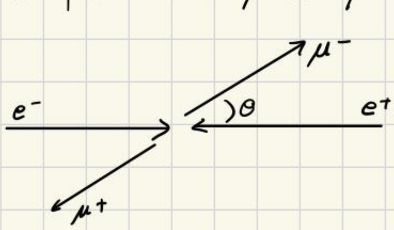
would be possible. and $\mu^- \longrightarrow e^-\gamma$ violates lepton flavor conservation.

Therefore from both the experimental and theoretical p.o.v. we have just 1 diagram.
What could happen is also the following:



with enough energy we can produce an intermediate state ($\sqrt{s} \geq m_X$) that then decays and produces $\mu^-\mu^+$.

The process $e^+e^- \longrightarrow \mu^+\mu^-$ is important because if we're able to make calculations with muons then this will open the way to quarks, substituting $\mu^+\mu^-$ with $q\bar{q}$.



$\underline{P}_{e^-} = (E_{e^-}, \vec{P}_{e^-}) = (E, \vec{P}_{in})$ $\qquad$ $\underline{P}_{\mu^-} = (E_{\mu^-}, \vec{P}_{\mu^-}) = (E, \vec{P}_{out})$

$\underline{P}_{e^+} = (E_{e^+}, \vec{P}_{e^+}) = (E, -\vec{P}_{in})$ $\qquad$ $\underline{P}_{\mu^+} = (E_{\mu^+}, \vec{P}_{\mu^+}) = (E, -\vec{P}_{out})$

N.B. we are assuming 2 symmetric beams $E_1 = E_2 \equiv E$.
The energy needed for the process is $\sqrt{s} \geq 2m_\mu$.

The expression for the differential cross section is:

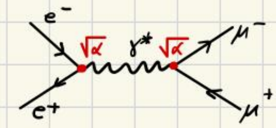
$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \cdot \frac{1}{(E_1+E_2)^2} \frac{|\vec{P}_{out}|}{|\vec{P}_{in}|} |\mathcal{M}|^2$$

If we compute it in the high energy limit $E_i \gg m_i \longrightarrow P_{out} \simeq P_{in}$. And if we use the Mandelstan variables $(E_1+E_2)^2 = s$. Therefore

$$\boxed{\frac{d\sigma}{d\Omega} \simeq \frac{1}{64\pi^2 s} |\mathcal{M}|^2}$$

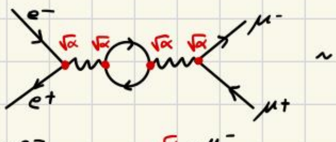
Let's now compute the matrix element $|\mathcal{M}|$. We have contributions from:

$\mathcal{M}_1$ =



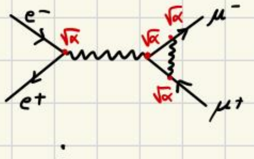
$\sim o(\alpha)$

$\mathcal{M}_2$ =



$\sim o(\alpha^2)$

$\mathcal{M}_3$ =

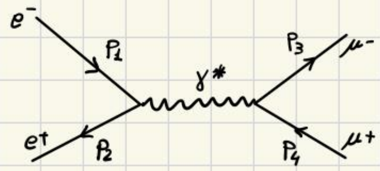


$\sim o(\alpha^2)$

$\vdots$

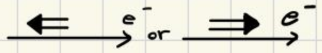
$$\left.\right\} \quad |\mathcal{M}|^2 = |\mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 + \dots|^2 = |\mathcal{M}_1|^2 + o(\alpha^3)$$

So we just focus on the first diagram (leading order). From the Feynman rule we compute:



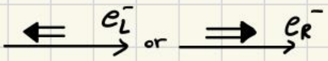
$$-i\mathcal{M} = \left[\bar{v}(P_2)\, ie\gamma^\mu u(P_1)\right] \times \left(\frac{-ig_{\mu\nu}}{q^2}\right) \times \left[\bar{u}(P_3)\, ie\gamma^\nu v(P_4)\right] = -\frac{e^2}{s} J_e^\mu J_{\mu\mu}$$

Now we have to take into account spin. Since we do not know spin we have to sum over all the possible configurations (usually the e⁻/e⁺ beams are not polarized). For the spin we have 2 possibilities it can either be up or down

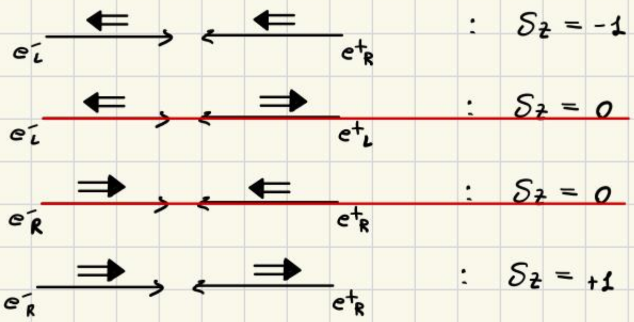


we then define the helicity as the projection of spin along the direction of motion: $h = \frac{\vec{S}\cdot\vec{P}}{|\vec{S}|\cdot|\vec{P}|}$

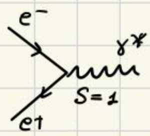
For a massless particle h is invariant. e⁻ is not massless but in the high energy limit can be considered massless. and it makes sense to consider left handed and right handend electrons:



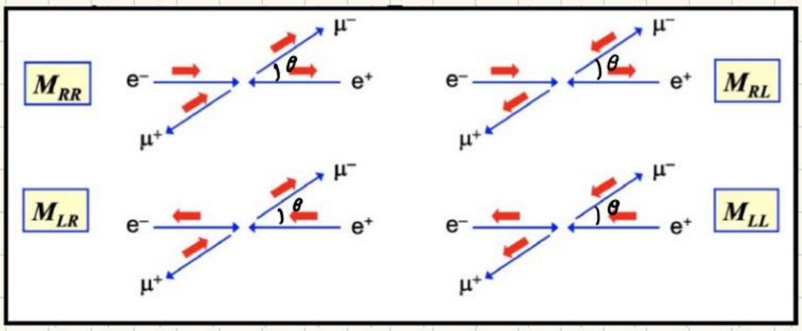
The possible configurations are:



} not possible because of ang. mom. conservation



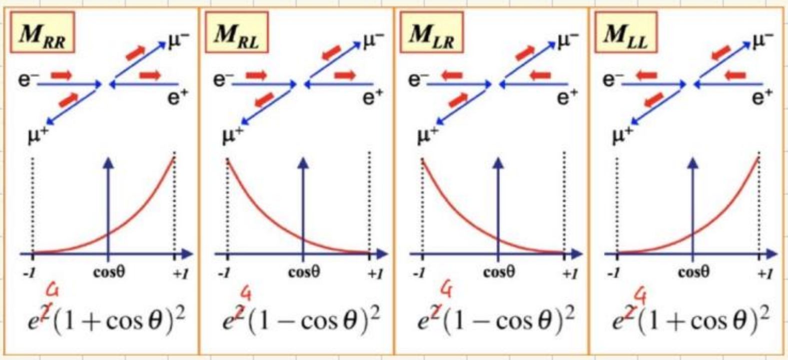
Analogously there are 2 possible final configuration states for μ and this makes up a total of 4 combinations. So what we have are the following configurations



M_{AB} means that we have e_A^- in the initial and μ_B^- in the final state

These are the 4 matrix elements we have to consider:

$$\boxed{\begin{aligned} |\mathcal{M}_{RR}|^2 &= |\mathcal{M}_{LL}|^2 = (4\pi\alpha)^2(1+\cos\theta)^2 \\ |\mathcal{M}_{RL}|^2 &= |\mathcal{M}_{LR}|^2 = (4\pi\alpha)^2(1-\cos\theta)^2 \end{aligned}}$$



$$\longrightarrow \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\cdot\frac{1}{s}\cdot\frac{1}{4}\left[|\mathcal{M}_{LL}|^2 + |\mathcal{M}_{RR}|^2 + |\mathcal{M}_{RL}|^2 + |\mathcal{M}_{LR}|^2\right] =$$

↳ we average

$$= \frac{1}{64\pi^2 s}\frac{(4\pi\alpha)^2}{4}\left[2(1+\cos\theta)^2 + 2(1-\cos\theta)^2\right] =$$

$$= \frac{\alpha^2}{16\, s}\left[4 + 4\cos^2\theta\right] =$$

$$= \frac{\alpha^2}{4s}\left[1+\cos^2\theta\right]$$

n.b. max for $\theta = 0$

$\xrightarrow[e^-]{} \xleftarrow[e^+]{} \Rightarrow \xleftarrow[\mu^+]{} \quad \xrightarrow[\mu^-]{}$

Now by integration we can obtain the total cross section:

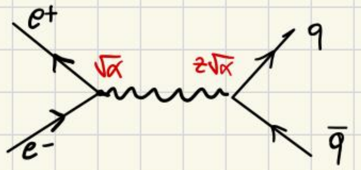
$$\boxed{\sigma_{tot} = \int \left(\frac{d\sigma}{d\Omega}\right) d\Omega = \frac{4\pi}{3}\frac{\alpha^2}{s}}$$

Let's convert it into something we can measure: s in GeV^2 and σ in barn:

- $\frac{4\pi}{3}\alpha^2 = 0.56 \cdot 10^{-4}$
- $\hbar c = 1 = 197$ MeV·fm $= 0.197$ GeV·fm $\longrightarrow 1\,fm = 10^{-15} m$, $1\,fm^2 = 10^{-30} m^2 = 10^{-26}$

$$\longrightarrow \sigma = 0.56 \cdot 10^{-4}\,\frac{(0.197)^2\, GeV^2 fm^2}{s} = \frac{21.6\, nb}{s\,[GeV]} = \frac{21.6\, nb}{4}\,\frac{1}{(E_{beam}[GeV])^2}$$

Now we can look at a process with quarks in the final state:



In QED all I care about is electric charge. QED is blind to color and all the rest.

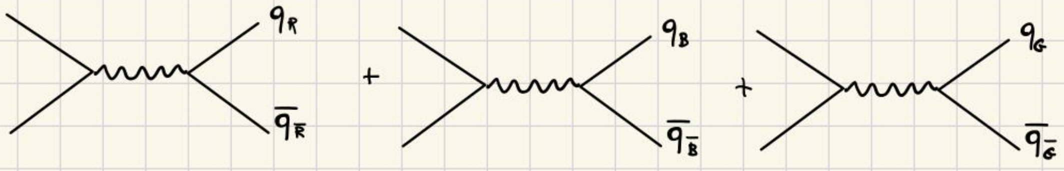
We take into account that quarks have a fractionary charge

$$q_q = Ze \quad , \quad Z = \begin{cases} +2/3 & u, c, t \\ -1/3 & d, s, b \\ 1 & \text{leptons} \end{cases}$$

This implies that:

$$\sigma(e^+e^- \longrightarrow q_c\bar{q}_c) \propto \sigma(e^+e^- \longrightarrow \mu^+\mu^-)\cdot Z_q^2$$

Since we don't know a priori which color the quarks have, we have to sum over the 3 possible configurations: $(N_c = 3)$



So we have:

$$\sigma(e^+e^- \longrightarrow q_c\bar{q}_c) \propto \sigma(e^+e^- \longrightarrow \mu^+\mu^-)\cdot Z_q^2 \cdot N_c$$

But this is for 1 flavor! How many flavors do we have? It depends on $\sqrt{s}$

$$\longrightarrow \boxed{\sigma(e^+e^- \longrightarrow q_c\bar{q}_c) = \sigma(e^+e^- \longrightarrow \mu^+\mu^-)\cdot N_c \sum_{\text{flavors}} Z_{q_i}^2}$$

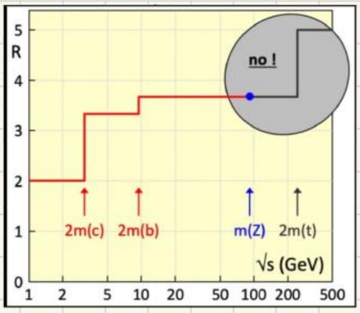
An important quantity is the ratio:

$$\boxed{R_{QED} = \frac{\sigma(e^+e^- \longrightarrow q\bar{q})}{\sigma(e^+e^- \longrightarrow \mu^+\mu^-)} = \frac{N(e^+e^- \longrightarrow q\bar{q})}{N(e^+e^- \longrightarrow \mu^+\mu^-)} = \sum_{\text{flavors}} Z_{q_i}^2 N_c}$$

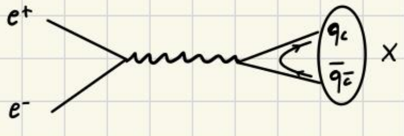
R is a function of $\sqrt{s}$:

- if $0 < \sqrt{s} < 2m_c$: only u, d, s: $R_{uds} = 3\left[\left(+\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2\right] = \frac{4+1+1}{9}\cdot 3 = 2$
- if $2m_c < \sqrt{s} < 2m_b$: also c : $R_{udsc} = R_{uds} + \left(\frac{2}{3}\right)^2\cdot 3 = 2 + \frac{4}{3} = \frac{10}{3}$
- if $2m_b < \sqrt{s} < 2m_t$: also b : $R_{udscb} = R_{udsc} + 3\cdot\left(-\frac{1}{3}\right)^2 = \frac{11}{3}$
- if $\sqrt{s} > 2m_t$: also t : $R_{udscbt} = 5$

Theoretically this ratio should be a step function:



In reality we don't have a step function because we could have some resonances X when $\sqrt{s} \sim m_X$



In that case the cross section becomes :

$$\boxed{\sigma(e^+e^- \longrightarrow X \longrightarrow f\bar{f}) = \frac{16\pi}{s}\,\frac{(2J+1)}{(2S_a+1)(2S_b+1)}\,\frac{\Gamma_{e^+e^-}}{\Gamma_{tot}}\,\frac{\Gamma_{f\bar{f}}}{\Gamma_{tot}}\,\frac{\Gamma_{tot}^2}{(m_X-\sqrt{s})^2 + \frac{\Gamma_{tot}^2}{4}}}$$

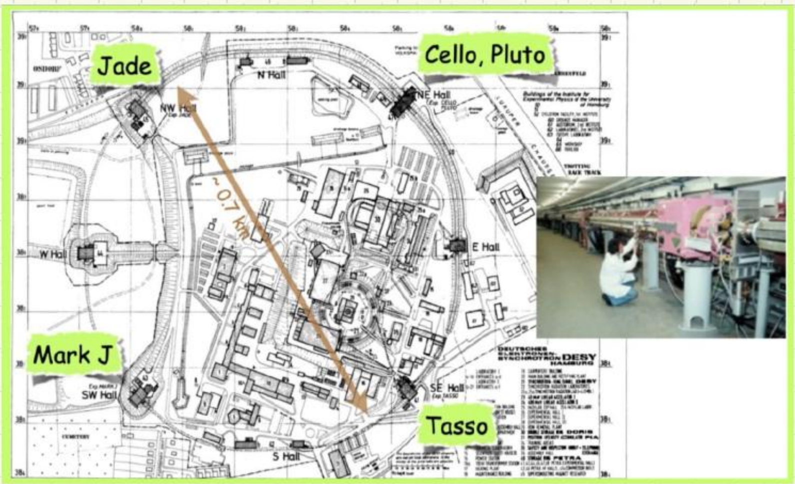
J: angular momentum of X

S_a, S_b: spin of incoming beams (in this case $S_a = S_b = 1/2$)

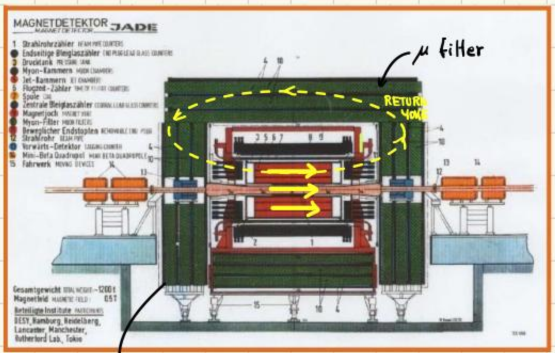
$\Gamma_{tot} = \Gamma_{e\bar{e}} + \Gamma_{f\bar{f}} + \Gamma_{ab} + \ldots$ (all decay channels)

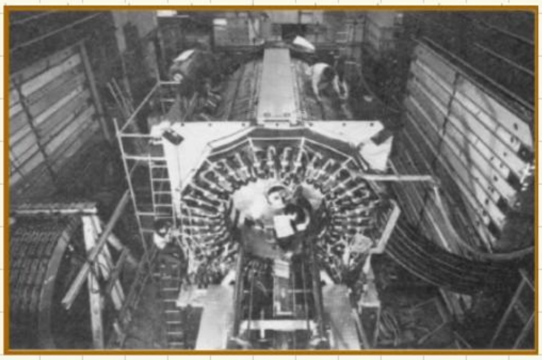
EXPERIMENTAL TESTS OF QED

The situation is that we have to find exp. evidence for all the building blocks that we inserted in the theory. Between 1978-1986 we had the e^+e^- collider Petra at Desy (near Hamburg) with different detectors



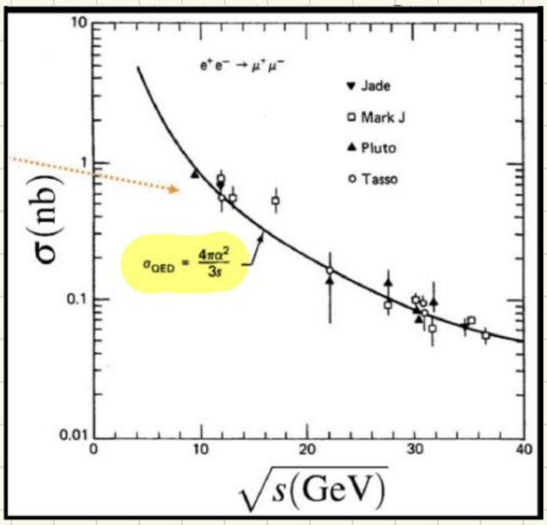
Between the detectors there was for ex. JADE (collab. Japan, Deutchland, England)





the structure of the detector is usually cilindrical because of the geometry of the $\vec{B}$: we want $\vec{B}$ = uniform inside and this is achievable with a cyl. geometry.

Putting together the data taken from all the experiments (Jade, Tasso, Cello, Mark J) was found the following result.



This makes sense with what we've already seen:

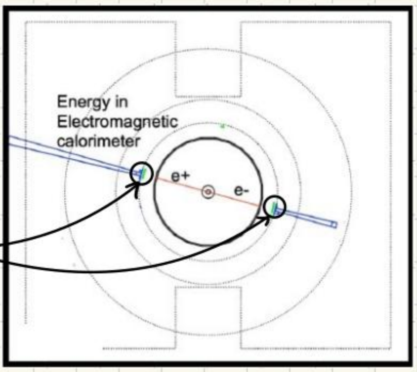
$$\sigma = \frac{21.6 \text{ nb}}{s\,[\text{GeV}^2]} \longrightarrow \sigma(\sqrt{s} = 10\text{ GeV}) \simeq 1\text{nb}$$

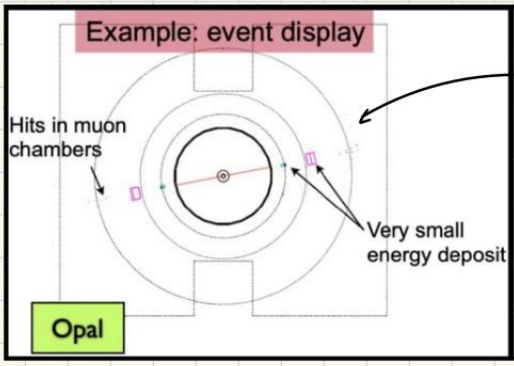
N.B. it is the total σ: the one that we obtain through:

$$N = \sigma \cdot \mathcal{L}_{inst} \cdot \Delta t$$

What we see, respectively in the case of $e^\pm$ and $\mu^\pm$ in the final states is shown in the 2 following figures:

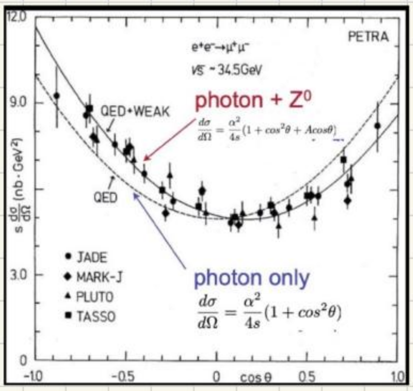
2 charged tracks + 2 large energy EM deposits



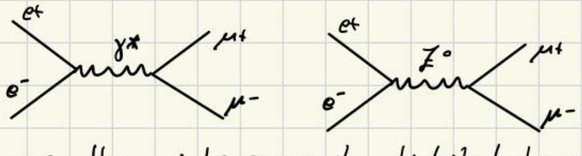


2 charged tracks + small EM deposit + signal in μ chambers

Then we can also look at the differential cross section, finding a trend not in agreement with what we expect $\frac{d\sigma}{d\Omega} \propto (1+\cos^2\theta)$ from QED. We find:

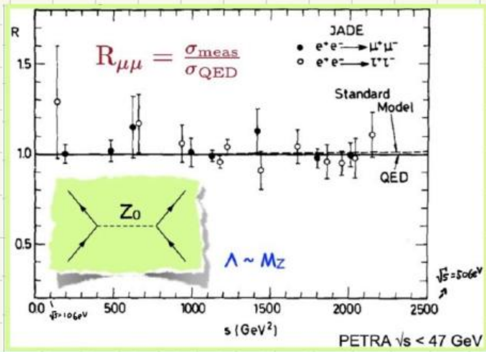


The point is that we only took into account γ as mediator. There is also Z^0:



As we can see the points are not distributed symmetrically and this tells that Z^0 give a contribution that does not go like $1+\cos^2\theta$

In order to see better the discrepancy we can plot the ratio between σ_{meas} and σ_{QED}:



As we can see after $\sqrt{s} \sim 50$GeV we start to see a discrepancy

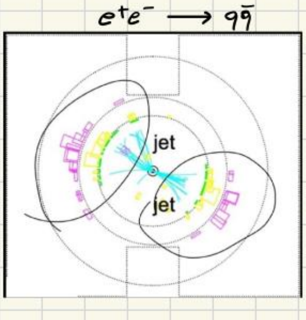
Beyond the e^+e^- and $\mu^+\mu^-$ channels we could also have $\tau^+\tau^-$ and hadrons in the final state. So to be more precise the total number of events is given by

$$N_{TOT} = \sigma_{tot} \times \mathcal{L}_{ins} \cdot \Delta t = (\sigma_1+\sigma_2+\sigma_3+\sigma_4) \times \mathcal{L}_{inst} \cdot \Delta t$$

where $\sigma_1, \sigma_2, \sigma_3, \sigma_4$ are the cross sections for the four different final states.

- The τ is so heavy that it has a lot of final states ($m_\tau = 1.78$ GeV) : $\tau^- \to e^-\bar{\nu}_e \nu_\tau$, $\tau \to 3\pi$, ...
- With hadrons what we see is the following:

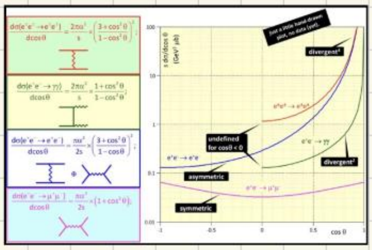
$e^+e^- \longrightarrow q\bar{q}$



Since we don't see single quarks in nature what happens is that quarks convert into hadrons (baryons and mesons) via a process called hadronization. Therefore what we see are the so called jets : bunches of stuffs all spread out, a mixture of everything (hadrons, e^+e^-, ...)

QED TESTS AND THE PRODUCTION OF QUARKS

The first question we want to look at is how to distinguish the t-channel from the s-channel



As we can see the 2 channels have different differential cross section (due to the configurations of spin). There is also an interference term and when we take the modulus square we see the sum of all of them.

We can test QED by looking at the angular distribution for a fixed energy and see that we get the expected behaviour. In the limit of small angles the behaviour is simpler

• limits of $d\sigma/d\cos\theta$ for $\cos\theta \to 1$ (i.e. $\theta \to 0$):

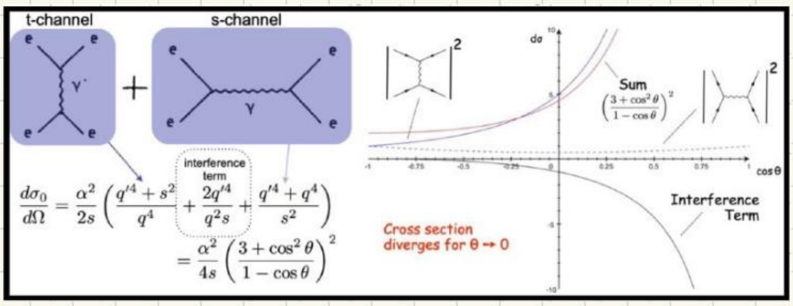
$$e^+e^+ \to e^+e^+: \quad \frac{2\pi\alpha^2}{s}\left(\frac{3+1}{\sin^2\theta}\right)^2 = \left(\frac{2\pi\alpha^2}{s}\right)\left(\frac{16}{\theta^4}\right);$$

$$e^+e^+ \to \gamma\gamma: \quad \frac{2\pi\alpha^2}{s}\,\frac{1+1}{\sin^2\theta} = \left(\frac{2\pi\alpha^2}{s}\right)\left(\frac{2}{\theta^2}\right);$$

$$e^+e^- \to e^+e^-: \quad \frac{\pi\alpha^2}{2s}\left(\frac{3+1}{2\sin^2(\theta/2)}\right)^2 = \left(\frac{2\pi\alpha^2}{s}\right)\left(\frac{16}{\theta^4}\right);$$

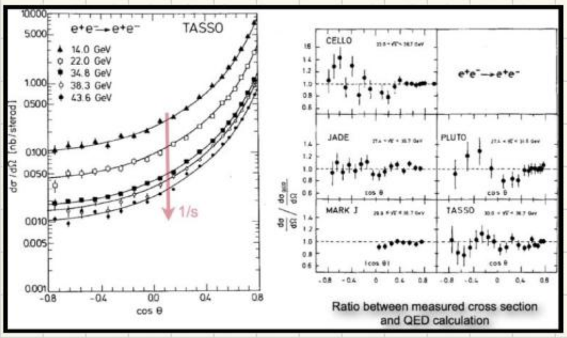
$$e^+e^- \to \mu^+\mu^-: \quad \frac{\pi\alpha^2}{2s}(1+1) = \left(\frac{2\pi\alpha^2}{s}\right)\left(\frac{1}{2}\right).$$

In the Bhabha scattering we have 2 channels and what we see is the sum of the amplitude squared + interference term:



To see whether we have everything under control or not we do a ratio plot (on the right):

We can also vary $\sqrt{s}$ and see that the angular distribution varies as expected.



By checking that the behaviour is the one expected we can look indirectly for new physics. In fact if Bhabha was:

$$\mathcal{M}_{Bh} = \mathcal{M}_t + \mathcal{M}_s + \mathcal{M}_x$$

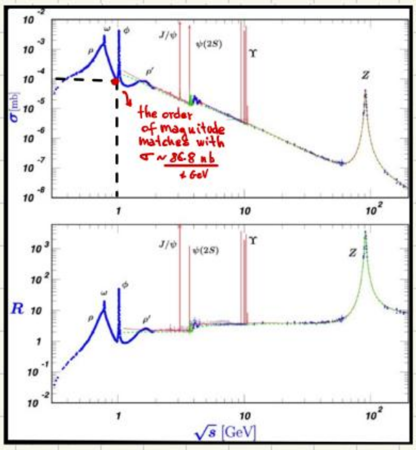
$$\longrightarrow \quad \sigma_{Bh} \propto |\mathcal{M}_t|^2 + |\mathcal{M}_s|^2 + 2\mathcal{M}_t^*\mathcal{M}_s + |\mathcal{M}_x|^2 + 2\mathcal{M}_x^*\mathcal{M}_t + 2\mathcal{M}_x^*\mathcal{M}_s$$

Now we're ready for $e^+e^- \to$ hadrons. The hadronization is there because quarks are not color singlet, hence they cannot be observed. What we measure are the baryons and mesons that the quarks made up. The only exception is the top quarks that is too heavy that it decays before making mesons and baryons. Our detector allows us to understand whether we have e^-, μ^-, hadrons (jets); then we look at the ratio:

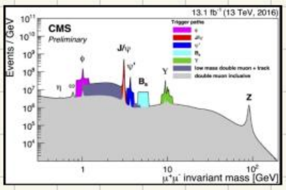
$$R = \frac{\sigma(e^+e^- \longrightarrow had)}{\sigma(e^+e^- \longrightarrow \mu^+\mu^-)}$$

and we plot it as $R = R(\sqrt{s})$. What we have is:

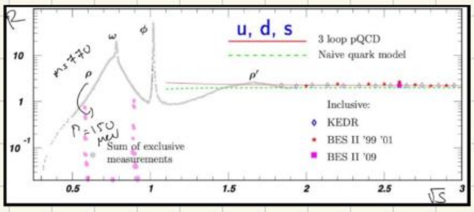
This plot is made up of many data points of different experiments (~ 40 years).



What we have here instead is the same plot with data taken by CMS in just 1 year (2016)



If we just look at the low energy region the behaviour is the following:

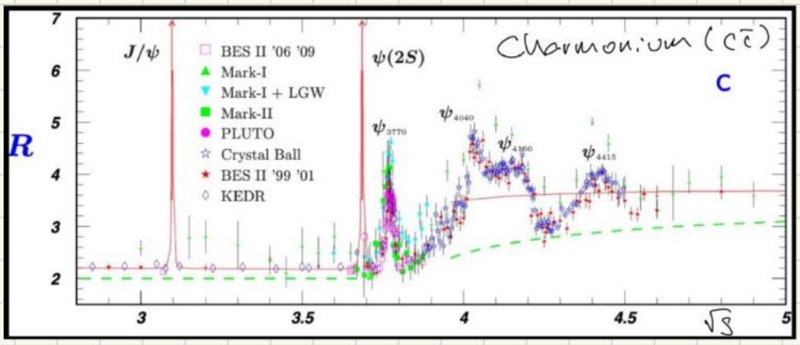


If $\sqrt{s} > 2m_d, 2m_u, 2m_s$ then $R \sim 2$ and indeed is what we see from the resonances. Near the resonances we see a specific Breit Wigner shape due to its decay width. We have that:

- $\rho(770)$: $\Gamma = 150$ MeV ; $\omega(787)$: $\Gamma = 8.7$ MeV ; $\phi(1020)$: $\Gamma = 4$ MeV ; $\rho'(1590)$: $\Gamma = 145$ MeV

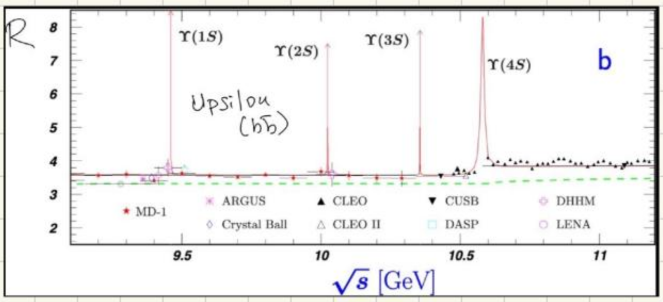
and in this case ϕ has the narower peak (and therefore long decay time)

So what we see in the plot is the flat spectrum + resonances. Without taking into account color we would be off by a factor of 3 that is a lot and so this is an indirect proof that color exists.
If we then increase the energy we can also produce charm quarks, and in the plot we start to see charmonium ($c\bar{c}$):



$$R_{usdc} = R_{uds} + \left(\frac{2}{3}\right)^2 \cdot 3 = 3 + \frac{1}{3}$$

Growing again the energy we start to produce b quarks and in the same plot we see also the upsilon Υ ($b\bar{b}$):



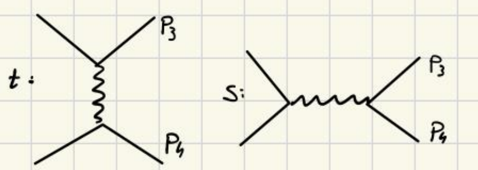
$$R_{udscb} = R_{udsc} + \left(-\frac{1}{3}\right)^2 \cdot 3 = 3 + \frac{2}{3}$$

So all these plots are the experimental proof that quarks exist and all we did was count the # of events:

$$R = \frac{\#(e^+e^- \longrightarrow \text{hadrons})}{\#(e^+e^- \longrightarrow \mu^+\mu^-)} \qquad (N = \sigma \cdot \mathcal{L}_{inst} \cdot \Delta t)$$

If we look at the t and s channel we have that:

t:



$$(P_3 + P_4)^2 = m_R = \sqrt{s}$$

In the t-channel we cannot have a resonance : $(P_3+P_4)^2$ cannot be a resonance because there is no correlation between P_3 and P_4. In the s-channel instead we have a resonance. So we could see whether we are in the t-channel or in the s-channel by looking at the angular distribution but in the case of s-channel we can also look at the invariant mass since we can have resonance looking resonances by measuring the invariant mass is a powerful tool for discoveries.

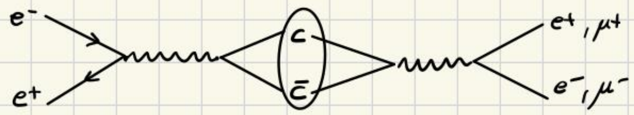
Measuring R is an indirect proof of the existence of something ; measuring a peak instead is a direct proof that something exists (because we're able to measure the mass and the decay width.

Let's now study the Discovery of Charmonium ($c\bar{c}$) in 1974. People were looking for new states through QED interactions. In 1970 the existence of a 4th quark was proposed via the GIM mechanism (developed to explain why there was a suppression of the decay $K^0 \to \mu^+\mu^-$).

2 different labs used 2 different techniques:

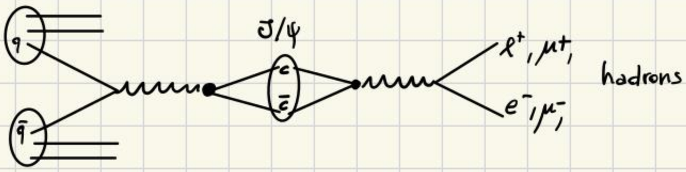
1) SPEAR (Stanford Positron Electron Asymmetric Ring) (Richter experiment)

$$e^+ + e^- \longrightarrow J/\psi \longrightarrow e^+e^- \dots$$



2) AGS (Alternating Gradient Synchrotron) used proton on a fixed target (Ting experiment)

$$p + Be \longrightarrow J/\psi \longrightarrow e^+, e^- \dots$$



Richter called its discovery ψ, Ting called it J , today we call it the J/ψ and it has:

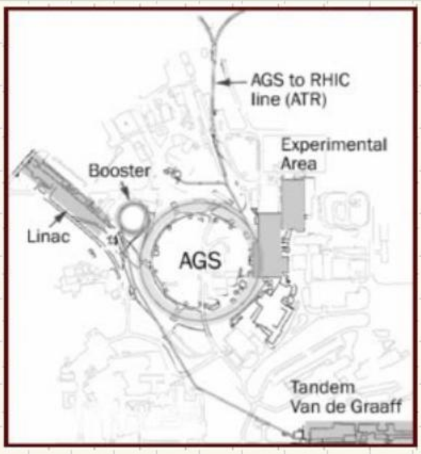
$$m = 3097 \text{ MeV} \; ; \quad \Gamma = 0.087 \text{ MeV} \; ; \quad \frac{\Gamma}{m} = 10^{-5} \quad \text{(extremely narrow)}$$

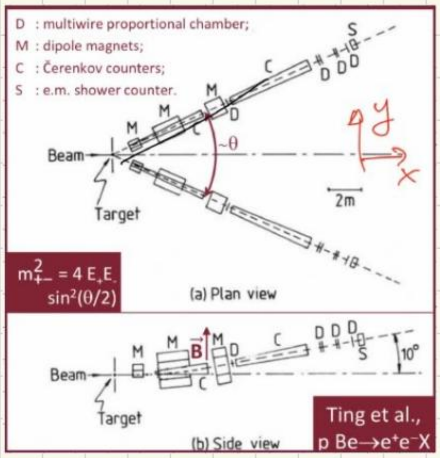
Let's now go through the Ting experiment $\quad P + Be \longrightarrow e^+ + e^- + X$

They looked at the invariant mass of e^+e^- in the final state. First of all we produce protons then we accelerate them linearly, then we accelerate them in the AGS and finally we smash them against a target. (10^{12} particles per pulse).

The detector is a 2 arms spectrometer. They fixed the angle at a certain value. They were dipole magnets (M) that let the particles follow the track. Then there was a Cherenkov detector (C). Then multiwire prop. chambers (D) (that leave a signal prop. to the energy of the particle. There were the EM shower counter (S) (to see whether we have an e^- or not).

Since the angle is fixed the magnets fix also the momentum so all the particles in (C) have the same momentum but different mass and so different velocities : this allows us to discriminate them.





Let's look at the Kinematics of the experiment

$$P^+ \equiv P(e^+) = (E^+, p^+\cos\tfrac{\theta}{2}, -p^+\sin\tfrac{\theta}{2})$$

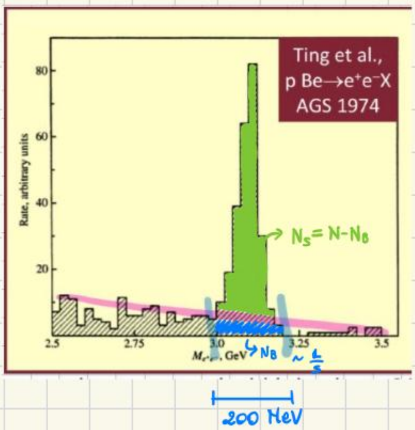
$$P^- \equiv P(e^-) = (E^-, p^-\cos\tfrac{\theta}{2}, +p^-\sin\tfrac{\theta}{2})$$

We are in the high energy limit : $p \gg m_e \longrightarrow p^\pm \simeq E^\pm$

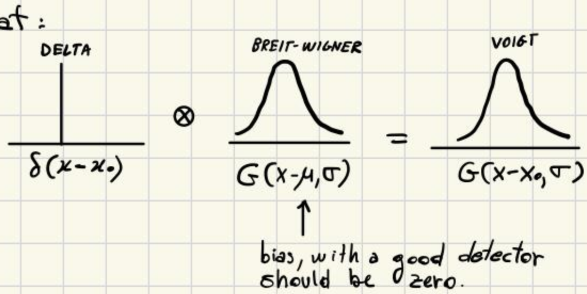
$$\longrightarrow (P^+ + P^-)^2 = P^{+2} + P^{-2} + 2P^+\cdot P^- = 2m_e^2 + 2(E^+E^- - \vec{p}^+\cdot\vec{p}^-) \simeq$$
$$\simeq 2E^+E^- - 2\vec{p}^+\cdot\vec{p}^- = 2E^+E^- - 2p^+p^-\cos\theta \simeq$$
$$\simeq 2E^+E^- - 2E^+E^-\cos\theta = 2E^+E^- - 2E^+E^-(\cos^2\tfrac{\theta}{2} - \sin^2\tfrac{\theta}{2}) =$$
$$= 2E^+E^-(1 - \cos^2\tfrac{\theta}{2} + \sin^2\tfrac{\theta}{2}) = 4E^+E^-\sin^2\tfrac{\theta}{2}$$

$$\longrightarrow \boxed{m(e^+e^-) = 4E^+E^-\sin^2\tfrac{\theta}{2}} \longrightarrow \text{we need to measure } E^+, E^-, \theta$$

N.B. If we had chosen $\mu^+\mu^-$ we would have had a worse resolution because μ goes into multiple scattering.
We care about res. because what we see is the convolution between the resolution and the Breit Wigner of the resonance. After the collision of many pulses the plot is:

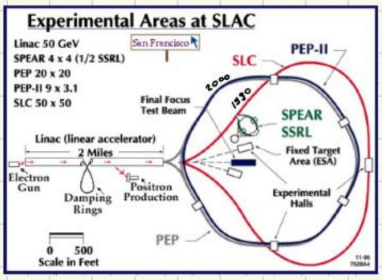


- Resonance at 3.0 – 3.2 GeV ; central value : $m = 3.1$ GeV
- Width : 0.2 GeV . Assuming it is 6 times the gaussin sigma we have : $\sigma \simeq 30$ MeV
- Remember that :



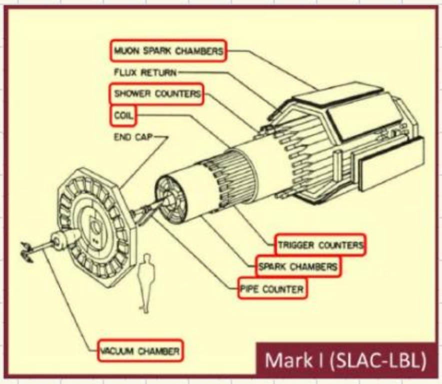
Doing a Gaussian fit and perform the de-convolution we can find the actual width of the particle
When Ting announced the result it called the particle the J and that is because the quantum numbers for this particle are the same of the photon $J^P = 1^-$

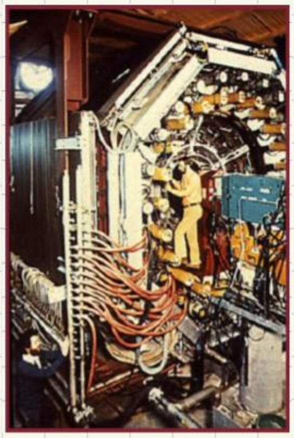
Let's now go through the Discovery of ψ at slac by Richter



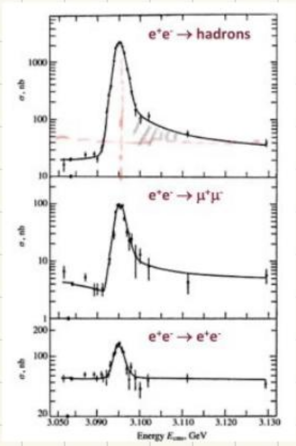
- e^+e^- collider with $\sqrt{s} = 2.5$ GeV $\longrightarrow 7.5$ GeV
- There is a LINAC used to accelerate e^+ and e^-
- When a larger energy was needed the radius was increased since this decreased the synchrotron radiation.

This was the synchrotron, let's now look at the detector:





This is the MARK-1 detector, similar to what we have today at CMS. In the exp. they were doing $e^- + e^+$ collisions scanning over different values of $\sqrt{s}$ in steps of 200 MeV. counting the # of events for each value. However with this step size they didn't see anithing. Reducing the step size to 2.5 MeV. they saw a clear resonance peak at 3.1 GeV



- The peak is asymmetric: it is larger on the left rather on the right
- There are more hadrons than leptons (this is because $\alpha_{strong} \sim 0.1 > \alpha_{EM} \sim \frac{1}{137}$

So we have a new particle and the idea was that it is a bound state $c\bar{c}$ where c is the new charm quark. They measured $m = 3.1$ GeV, hence taking into account also the binding energy of strong interaction we can infer the mass of the charm quark:

$$m_{c\bar{c}} = 2m_c - B \longrightarrow m_c = \frac{3.1}{2} + B \simeq 1.5 - 1.6 \text{ GeV}$$

We know that

$$\sigma(e^+e^- \longrightarrow J/\psi \longrightarrow f\bar{f}) = \frac{2J_R+1}{(2\cdot\frac{1}{2}+1)(2\cdot\frac{1}{2}+1)} \cdot \frac{4\pi}{|\vec{P}_{in}|^2} \frac{\Gamma_{ee}}{\Gamma_{tot}} \frac{\Gamma_{f\bar{f}}}{\Gamma_{tot}} \frac{\Gamma_{tot}^2}{(\sqrt{s}-m_R)^2 + \frac{\Gamma_{tot}^2}{4}}$$

- The resonance R is the J/ψ, so:

$$J_R = 1 \longrightarrow \frac{2J_R+1}{(2\cdot\frac{1}{2}+1)(2\cdot\frac{1}{2}+1)} = \frac{3}{4}$$

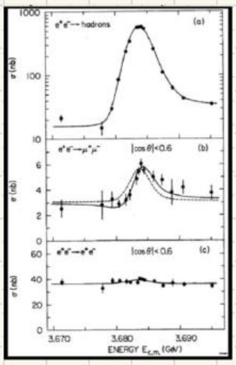
• In the relativistic regime: $|\vec{P}_{in}| \simeq E \longrightarrow s = 4E^2 \simeq 4|\vec{P}_{in}|^2 \longrightarrow \frac{4\pi}{|\vec{P}_{in}|^2} \simeq \frac{16\pi}{s}$

$$\longrightarrow \sigma(e^+e^- \longrightarrow J/\psi \longrightarrow f\bar{f}) = \frac{3}{4}\frac{16\pi}{s}\Gamma_{ee}\Gamma_{f\bar{f}}\frac{1}{(\sqrt{s}-m_{J/\psi})^2+\frac{\Gamma_{tot}^2}{4}}$$

We can denote:
- Γ_{ee} : $\Gamma(J/\psi \to e^+e^-)$
- $\Gamma_{f\bar{f}}$: $\Gamma(J/\psi \to f\bar{f})$ $\qquad f \in \{\mu, q\}$

and our assumption is : $\Gamma_{tot} = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{had}$ $*$

So there are 4 unknows $\Gamma_{ee}, \Gamma_{\mu\mu}, \Gamma_{had}, \Gamma_{tot}$ but we can measure $\sigma_{ee}, \sigma_{\mu\mu}, \sigma_{had}$ and using $*$ we also know Γ_{tot}. So there are 4 unknows with 4 eq. The Γ_{tot} as we know it today is $\Gamma_{tot} = 0.087$ MeV. The width is so narrow that this resonance is practically a δ.

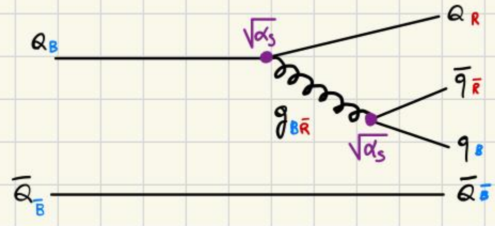


In the Richter exp. they continued to take data and 10 days later they had another peak. That is very similar to the previous one but at a greater energy : it has been called ψ'

Decay of $Q\bar{Q}$ resonances

Let's consider a particle made by $Q\bar{Q}$ where $Q = s, c$. e.g. $\phi \sim s\bar{s}$ $J/\psi = c\bar{c}$.
Let's look at the strong decay of $Q\bar{Q}$:

$$Q\bar{Q} \longrightarrow Q\bar{q} + \bar{Q}q$$



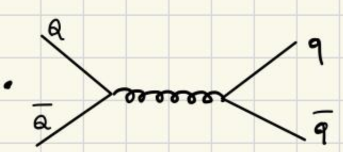
$\phi(s\bar{s})$ DECAY $\quad \Gamma = 4$ MeV

$\phi(1020)$ DECAY MODES

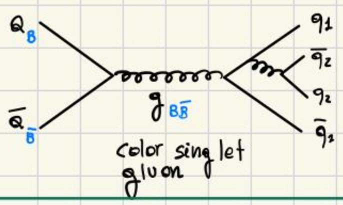
	Mode	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1	K^+K^-	(49.1 ± 0.5) %	S=1.3
Γ_2	$K_L^0 K_S^0$	(33.9 ± 0.4) %	S=1.2
Γ_3	$\rho\pi + \pi^+\pi^-\pi^0$	(15.4 ± 0.4) %	S=1.2
Γ_4	$\rho\pi$		
Γ_5	$\pi^+\pi^-\pi^0$		
Γ_6	$\eta\gamma$	(1.301 ± 0.025) %	S=1.2
Γ_7	$\pi^0\gamma$	$(1.32 \pm 0.05) \times 10^{-3}$	
Γ_8	$\ell^+\ell^-$	—	

the most likely decay channels are:
$s\bar{s} \longrightarrow s\bar{u} \quad \bar{s}u \qquad (K^- K^+)$
$s\bar{s} \longrightarrow s\bar{d} \quad \bar{s}d \qquad (\bar{K}^0 K^0)$

What about the annihilation channel?

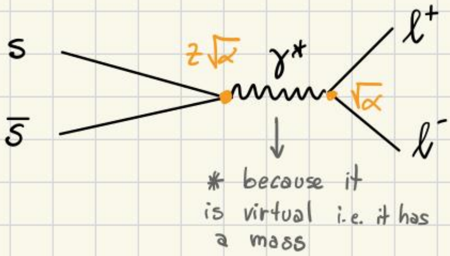


e.g. $\phi \longrightarrow \pi^0$ (1-body decay) violates E-mom conservation



e.g $\phi \longrightarrow \pi^+\pi^-$ we have 8 colored gluons. It's not possible to have a colorless gluon

How to produce $\ell^+\ell^-$ in the final state? (As we can see from the table)



Why the branching ratio is so small?

We Know that : $\Gamma(a \longrightarrow bc) \propto |\mathcal{M}(a \to bc)|^2 \rho(E_f = E_i)$.

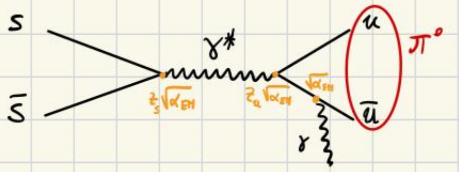
- $\phi \longrightarrow K^+K^-$ $\quad m_K \simeq 495$ MeV , $m_\phi \simeq 1020$ MeV $\longrightarrow Q \sim 1020 - 2\cdot 495 = 30$ MeV
- $\phi \longrightarrow \ell^+\ell^-$ $\quad m_e \simeq 0.5$ MeV $\longrightarrow Q \sim 1020 - 1 \sim 1019$ MeV

$$\mathcal{M}_{EM} \sim Z\sqrt{\alpha_{EM}} \cdot \sqrt{\alpha_{EM}} = Z\,\alpha_{EM} = \frac{Z}{137}$$

$$\mathcal{M}_{QCD} \sim \sqrt{\alpha_S}\cdot\sqrt{\alpha_S} = \alpha_S = 0.1 \qquad \longrightarrow \qquad \Gamma_{EM} \leq \frac{1}{100}\,\Gamma_{strong}$$

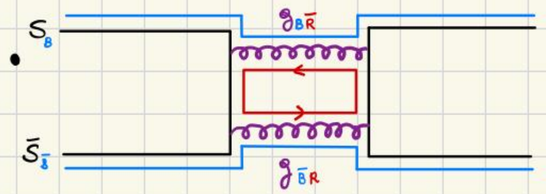
Hence we understand why it is so small.

How to produce $\pi^0\gamma$ in the final state?



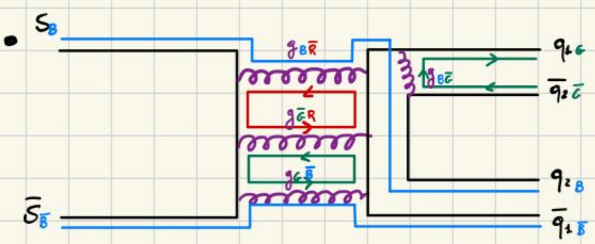
$$\pi^0 = \frac{u\bar{u} - d\bar{d}}{\sqrt{2}} \qquad\qquad \mathcal{M} \sim z_s z_u \alpha_{EM}^{3/2}$$

The other decays left are all hadronic processes

- 

$Q\bar{Q} \longrightarrow 2$ gluons

- C parity : violated $\quad \begin{cases} C(s\bar{s}) = -1 \\ C(gg) = [C(g)]^2 = +1 \end{cases}$

- 

- e.g: $\phi \longrightarrow \pi^+\pi^-$; $\phi \longrightarrow \pi^0\pi^0$ $\quad$ (n.b. $q_1, q_2 \in \{u, d\}$)
- C parity : conserved
- G parity $= C \times I_2^{rot}$ must be conserved!
- P parity must be conserved

We Know that

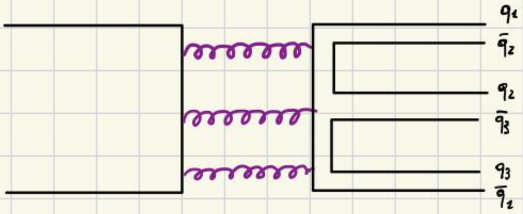
$$\begin{cases} \phi : & J^{PC} = 1^{--} \text{ and } I^G = 1^- \quad (\text{n.b. } G = (-1)^{I+S} = -1). \\ \pi^\pm : & J^P = 0^- \text{ and } I^G = 1^- \\ \pi^0 : & J^{PC} = 0^{-+} \text{ and } I^G = 1^- \end{cases}$$

Therefore:

	ϕ	$\longrightarrow \pi^+\pi^-$		
P	(-1)	(-1)(-1)	violated	$\longrightarrow$ not allowed
G	(-1)	(-1)(-1)	violated	

$$\phi \longrightarrow \pi^0 \pi^0 \qquad \longrightarrow \text{not allowed}$$
$$G \ (-1) \qquad (-1)(-1) \qquad \text{violated}$$

We need an other particle in the final state:

- 

- e.g. $\phi \to \pi^+ \pi^- \pi^0$ (n.b $q_1, q_2, q_3 \in \{u, d\}$)

This can be possible if for example $\phi \longrightarrow \rho\, \pi^0$ and then $\rho \longrightarrow \pi^+ \pi^-$

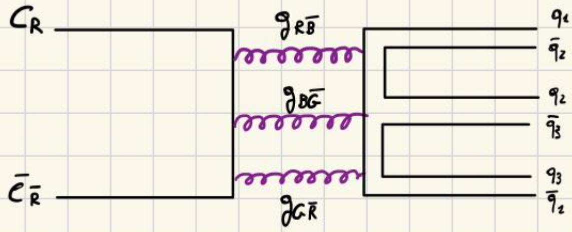
J/ψ DECAY $\Gamma = 87$ KeV

J/ψ(1S) DECAY MODES

	Mode	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
Γ_1	hadrons	(87.7 ± 0.5) %	
Γ_2	virtual$\gamma \to$ hadrons	(13.50 ± 0.30) %	
Γ_3	ggg	(64.1 ± 1.0) %	
Γ_4	γgg	(8.8 ± 1.1) %	
Γ_5	e^+e^-	(5.971 ± 0.032) %	
Γ_6	$e^+e^-\gamma$	[a] (8.8 ± 1.4) $\times 10^{-3}$	
Γ_7	$\mu^+\mu^-$	(5.961 ± 0.033) %	

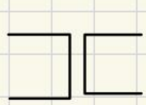
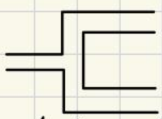
$\longrightarrow$ Why $J/\psi \to D^+ D^-$ ($D^0 \bar{D}^0$) is not possible? Because $m_{J/\psi} = 3.1$ GeV, $m_D = 1.864$ GeV $\longrightarrow m_{J/\psi} < 2\, m_D$

The C parity conservation allows us to have only strong decays with at least 3 gluons:



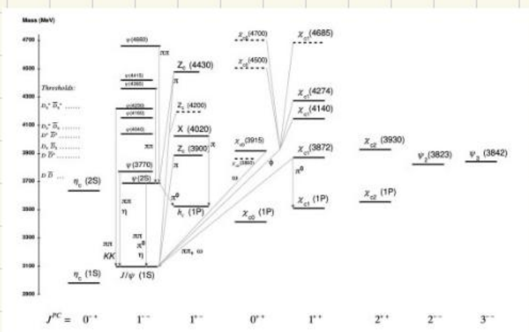
We use $q_1, q_2, q_3 \in \{u, d\}$ (not s because it is too heavy)

We could go on and add more gluons and more quarks until we have space (until Qvalue allows it). This type of decay (with 3 gluons) has a $\mathcal{M} \sim \alpha_s^3$ (heavily suppressed); this also explain why J/ψ is so narrow. At that time this fact was not know and there was an empirical rule: the OZI RULE that said that:

 is suppressed ;  is favored

OZI stands for (Okubo-Zweig-Iizuka) 1966. Later when QCD was estabilised they explained the rule theoretically.

N.B. Of course with a greater J we have more wide invariant masses so J/ψ(2S) is 3 times wider than the J/ψ(1S).



DISCOVERY OF τ LEPTON

What we have so far is :
$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix} \quad \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}$$
$$\begin{pmatrix}u\\ d\end{pmatrix} \quad \begin{pmatrix}c\\ s\end{pmatrix}$$

The discovery of a new more particle would imply the existence of a whole new family (leptons and quarks). This discovery came in 1976 by SLAC with MARK1 (Nobel prize 1995).
He was looking at events of the kind :

$e^+e^- \longrightarrow e^+\mu^-$ (unbalanced, i.e. there is something else other than the electron and the muon)

So:

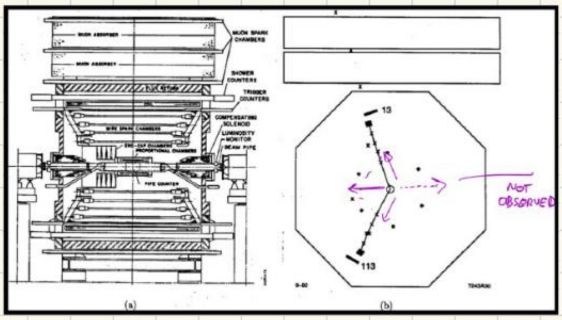
- $e^+e^- \longrightarrow X^+X^-$, $X^- \to e^- + X$, $X^+ \to \mu^+ + X$

Other types could be:

$e^+e^- \longrightarrow e^+e^-$ (unbalanced)
$e^+e^- \longrightarrow \mu^+\mu^-$ (unbalanced)

(They have more BG)

The detector was the following :

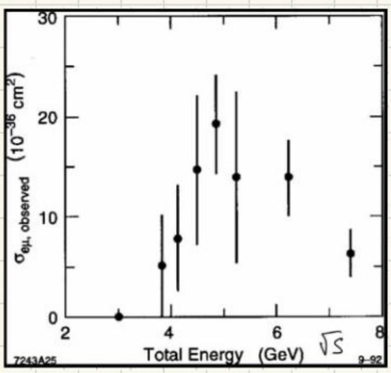


- μ passes through the iron and still leaves a signal after while instead e^- leaves all its energy inside the ECAL.
- As we can see on the right, the decay is unbalanced, that is the $\sqrt{s}$ does not add up. We see no other track and it should be to account for the missing 4 momentum

What they said is that they were neutrinos. The process that actually happens is the following :

$$\boxed{e^+e^- \longrightarrow \tau^-\tau^+,\quad \tau^+ \to \mu^+\nu_\mu\bar{\nu}_\tau,\quad \tau^- \to e^-\bar{\nu}_e\nu_\tau}$$

If this τ is heavy we cannot produce it if we are below treshold ; to have the production of unbalanced $e^-\mu^+$ ($e^+\mu^-$) the treshold is $\sqrt{s} \geq 2m_\tau$.
In the experiment they saw:



From this plot we clearly see that we have a treshold production that tells us that

$$\boxed{m_\tau \simeq 1.8\ GeV}$$

This is the proof of the existence of a new particle, the τ.

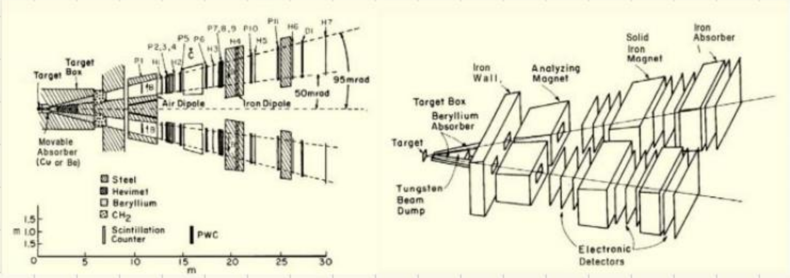
How do we know τ is a lepton? It interacts exactly as e^- and μ do.

Discovery of b quark

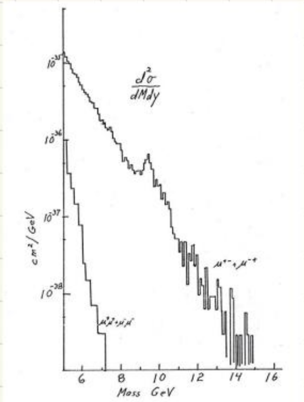
In 1977 Leon Lederman at Fermilab discovered the b quark (Nobel prize 1988). He used a 400 GeV proton beam on a target (10^{11} particles per pulse)

$$P + Cu\,(Pt) \longrightarrow \mu^+\mu^- + X$$

The detector was a 2 arms spectrometer :

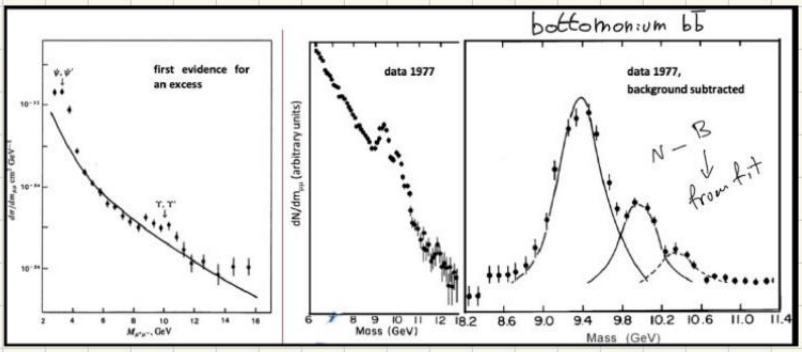


Combining muons with the same charge what they found was the following:



- The $\mu^+\mu^+$ ($\mu^-\mu^-$) channel has as expected any peak (this was a proof that they had the BG under control.
- In the $\mu^+\mu^-$ channel they saw an excess of events that cannot be explained by known particles

The same data can be also shown in the following plots:



the plot on the right is a BG subtracted plot : to obtain it we start from the observed plot that can we model as background + gaussin peak

$$\text{Signal} = \text{BG} + \text{Gaussian}$$

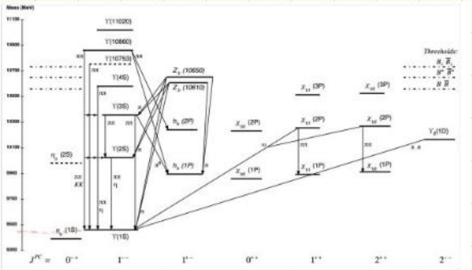
In our case we have a falling BG that we can model as a decreasing exp. $BG = Ae^{-\alpha x}$ and then 3 peaks that we model with 3 gaussians . So the signal can be fitted with:

$$\text{Signal} = Ae^{-\alpha x} + B \cdot G(x_1,\sigma_1) + C \cdot G(x_2,\sigma_2) + D \cdot G(x_3,\sigma_3)$$

Once the fit is done we have a best fit curve of the BG. So we can subtract it from the actual signal and obtain a background subtracted plot with only the peak.

What they saw is the convolution of a Breit-Wigner and a Gaussian resolution. The fact that the Gaussian well fit the data is due to the fact that the detector resolution dominate over the intrinsic Breit-Wigner.

They called the found bound state bottomonium. Later in 1980 at Slac, Cesr, Cornell, PEP-II, KEK did a scan and what they observed was:



We're seeing the family of $b\bar{b}$ states, that they called Upsilon $\Upsilon = (b\bar{b})$

As we can see there are 4 peaks: the 1st is the narrower which means that it represents the more stable bound state

Also in this case we have a whole spectroscopy (as the charmonium, positronium ecc.) with the only difference that the potential of the bound state and the masses change.

In a while open B states were discovered as: $B\ (b\bar{q})$, $B_s\ (b\bar{s})$, $B_c\ (b\bar{c})$, $B^0 (b\bar{u})$, $B^+ (b\bar{d})$ with $m_B = 5.279$ GeV.

The most probable decays for Υ would be:

$$b\bar{b} \longrightarrow b\bar{u}\ \bar{b}u\ (B^0\bar{B}^0) \quad ; \quad b\bar{b} \longrightarrow b\bar{d}\ \bar{b}d\ (B^-B^+)$$

However this is not possible for the $\Upsilon(1S)$ or $\Upsilon(2S)$ or $\Upsilon(3S)$ but becomes possible for the $\Upsilon(4S)$ since:

$$m(\Upsilon(4S)) = 10.580\ \text{GeV}, \quad m(B) = 5.279\ \text{GeV} \longrightarrow Q_{value} = m(\Upsilon(4S)) - 2m(B) > 0$$

So the $\Upsilon(4S)$ produces with an high probability $B^0\bar{B}^0$ and it is used as B factory (made with an e^+e^- collider at the energy $\sqrt{s} = m(\Upsilon(4S))$.

So at this moment of the last family we know:

$$\begin{pmatrix} \\ b \end{pmatrix} \begin{pmatrix} \\ \tau^- \end{pmatrix}$$

we miss the ν_τ and the "top" quark.

Discovery of top quark t

The idea was to look for something like:

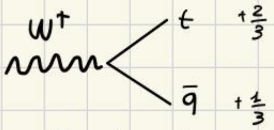
$$e^+e^- \longrightarrow t\bar{t} \longrightarrow \text{final states} \qquad \sqrt{s} \geq 2m_t$$

$$p + \bar{p} \longrightarrow t + \bar{t} + X$$

$$p + p \longrightarrow t + \bar{t} + X$$

However there weren't signes of top in $p\bar{p}$ collisions ($Sp\bar{p}S$ and Fermilab) and e^+e^- collisions (at LEP) (Scanning $\sqrt{s}$: $90 \longrightarrow 100$ GeV) (there was a theoretical idea of m_Z).

The best way to discover top quark is through hadronic machines in weak interactions: m_{top}, m_W, m_Z are related. The study of W, Z bosons provides constraints on m_t. In fact in 1983 it was discovered the W and Z and what it was understood is that $m_t \geq m_W, m_Z$. They had infact the following process:



$\longrightarrow q = b, s, d$

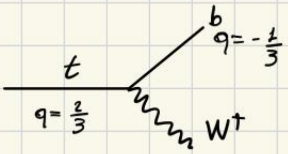
As we'll see weak interactions prefer to stay in the same family, so the most likely process to happen is:

$$W^+ \longrightarrow t\bar{b}$$

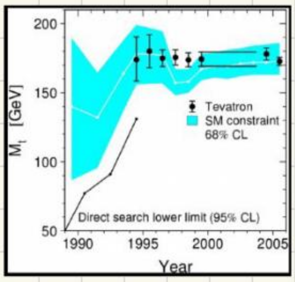
this implies that $m_W > m_t + m_b$. From Lederman we can infer that $m_\Upsilon \simeq 10\,GeV \longrightarrow m_b \simeq 5\,GeV$
In 1983 we discovered $m_W \simeq 80$ GeV $\longrightarrow$ we can put limits on the top mass. If the top was lighter than W it should have been already discovered but since is not the case we can conclude that:

$$\boxed{m_{top} > m_W}$$

So if the top is heavier than the W the process that we should look for is the top decay not the W decay:

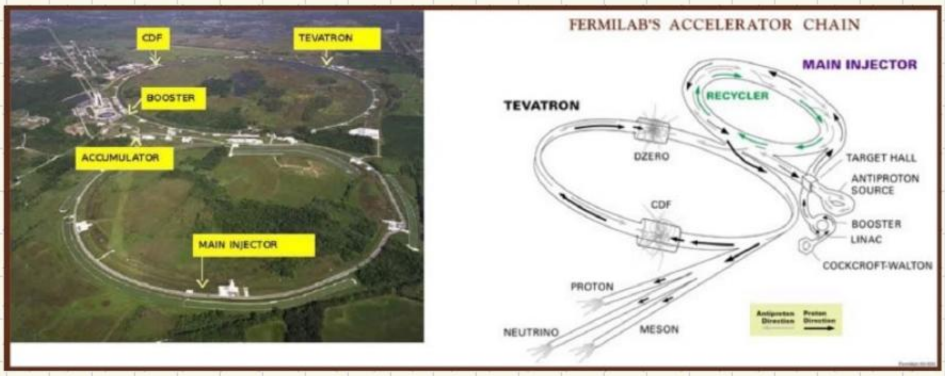


From LEP and from calculation we had a prediction on the top mass:



- The mass goes from 100 to 300 GeV (not easy for e^+e^- colliders)
- So the best thing to do is to use strong interactions and not QED to produce the top. So the best was to use an hadron collider.

In 1994 they had evidence of where the top mass should be with the Tevatron at Fermilab.

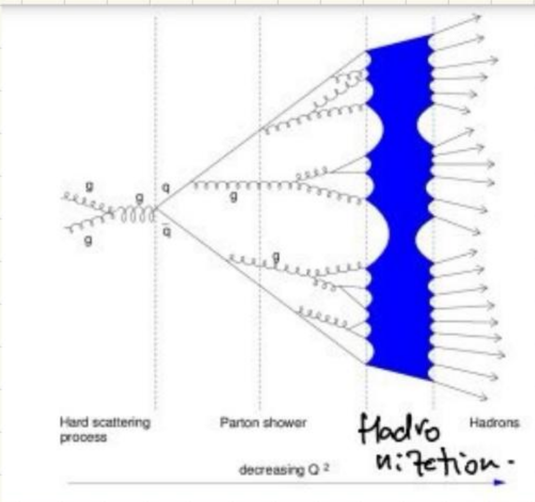


- Larger version of $Sp\bar{p}S$ with $\sqrt{s} = 1.96$ TeV
- We can either do a colliding beam experiment or a fixed target experiment at DZERO or CDF detectors.

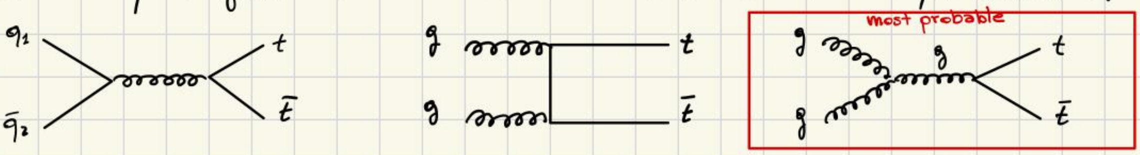
Let's look at how to produce top quark. Top quark interacts strongly so we look at strong interactions

$$p + \bar{p} \longrightarrow t + \bar{t} + X$$

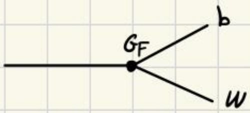
Here, due to the nature of strong interactions (and of hadronization) we have a lot of final states. On a time scale of $\sim 10^{-23}$ s the hadronization process happens: a quark emits a gluon then other quarks and from a shower of quarks and gluons we end up with hadrons:



There are many diagrams in QCD that can contribute to the production of 2 top :



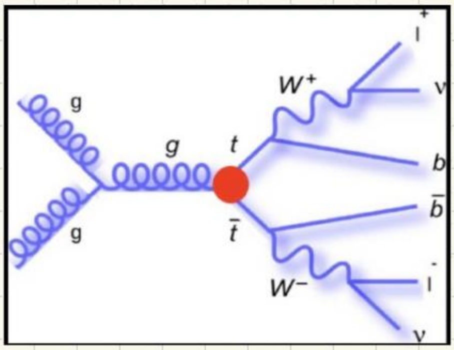
What is peculiar about the top is that there are no top mesons ($t\bar{q}$) or top baryons (tq_1q_2) and there is no toponium ($t\bar{t}$) bound state. That is because the top is so heavy that it decays before forming a bound state. From Fermi theory we found that:



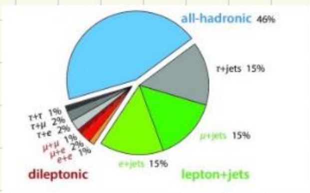
$\Gamma \simeq G_F m_t^3$ $\quad m_t \simeq 1.75\,\text{GeV}$, $\; G_F = 1.16 \cdot 10^{-5}\,\text{GeV}^{-2} \longrightarrow \Gamma \simeq 2\,\text{GeV} \longrightarrow \tau = \frac{1}{\Gamma} \sim 10^{-25}\,s$

this is obtained looking at weak interactions. Strong interactions have a characteristic time of $\tau \sim 10^{-23} s$ so this means that the top decays even before interacting at 100% $t \longrightarrow bW$.

As we said the most probable thing to happen is the g-g fusion :



- We can have many final states $W^+ \longrightarrow \ell^+\nu_e$, $W^+ \longrightarrow q_1\bar{q}_2$
- Then we have to dress the final states containing quarks with hadronization



The W can decay in many ways but we'll understand the simpler one is the one with leptons $W \to \ell\nu_e$

$$p\bar{p} \longrightarrow t\,\bar{t}, \quad \bar{t} \longrightarrow W^- + b, \quad t \longrightarrow W^+ + \bar{b}$$

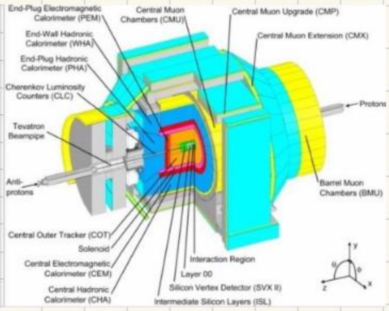
- for each b we have a jet of hadrons
- if the W decays into $q_1\bar{q}_2$ we have 2 more jets of hadrons
- if W decays into $\ell\nu_e$ we do not have other jets.

So we have a lesser messy situation with $W \to \ell\bar{\nu}_e$ because that is the situation with the smaller number of jets of hadrons around. So the golden mode is:

$$\boxed{p + \bar{p} \longrightarrow \underbrace{2\text{ jets}}_{\substack{\text{signals into}\\ \text{ECAL and}\\ \text{HADROCAL}}} + \underbrace{\ell_1^+ + \ell_2^-}_{\substack{\text{we expect tracks}\\ \text{into ECAL}}} + \underbrace{\text{missing energy}}_{\text{neutrinos}}}$$

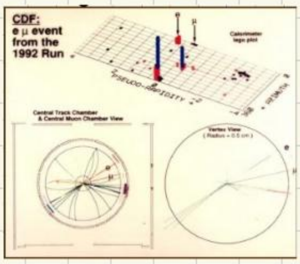
N.B. Although $W \to e\nu_e$ is the most clean channel it's B.R is only ~10%. So we need to take into account that the expected number of events is the 10% of the total.

Let's now have a look at the CDF experiment:



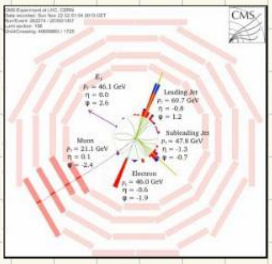
- Same geometry of CMS. (but lower res.)
- CDF has a gas tracker (instead now CMS and ATLAS have silicon tracker)

By looking at the invariant mass we can see top:



- Here we have a lego plot: there are 2 parameters (x, y) and the energy (z)

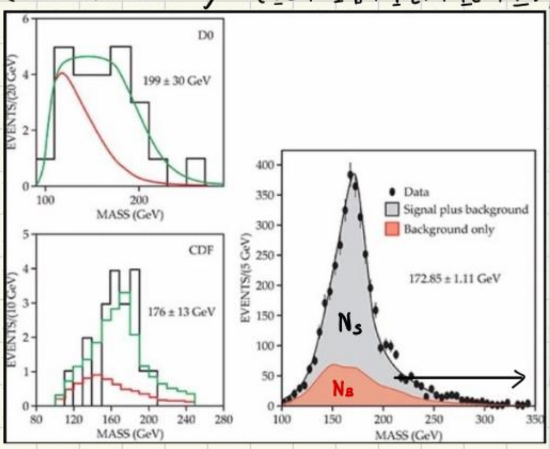
Nowadays an event look like this:



- What change is the granularity that is much finer today

The plot of the discovery was the following:

(Invariant mass of $(P_b + P_{\bar{b}} + P_{e^+} + P_{e^-} + P_\nu)^2$)



$N_S = N_{tot} - N_B$

- D0 had a worse res than CDF. This is due to the presence of ECAL and no trackers in D0.
  - $\frac{\sigma(ECAL)}{E} \sim 1/\sqrt{E}$ (better res with greater E)
  - $\frac{\sigma(tracker)}{p} \sim p$ (better res with lesser p, because it's more easy to bend with $\vec{B}$)
- Combining the infos from the 2 exp. they were able to found the top, as shown in the plot.

The only thing we miss is ν_τ but to understand its discovery we first need to understand weak interactions.

Weak interaction

Number of neutrino flavors
From Fermi model to V-A theory
Applications of weak interactions: Muon-decay, Pion-decay
Lepton flavor universality
Weak interactions of hadrons
Weak interactions in quarks
Flavor changing evidences
Cabibbo angle
GIM mechanism
Neutral weak current
Modern Electro-weak theory
Discovery of W and Z

Goldhaber experiment

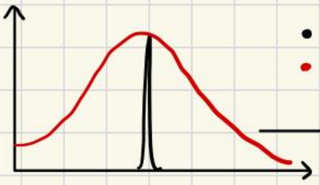
WEAK INTERACTIONS

Weak interactions were first observed in the slow process $\tau \sim 10^{-6}/10^{-8}\,s$ of nuclear β-decay. The theory of weak int. was born in parallel with the evidence of the existence of ν.

$$ {}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + e^- + X $$

$\Downarrow$ X has been called neutrino

$$ {}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + e^- + \bar{\nu}_e $$

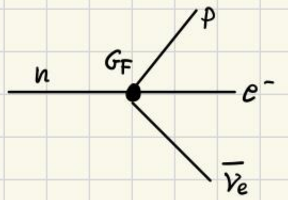


- expected
- observed

$\longrightarrow$ experimental evidence of a 3 body decay

The 1st theory proposed to describe the nuclear β-decay was the Fermi-theory. In the Fermi theory the interaction is pointlike and described by a 4-fermion coupling $G_F = 1.16 \cdot 10^{-5}\,GeV^{-2}$ (n.b. notice the difference between α_{QED} and G_F : α_{QED} is dimensionless)

$$ {}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + e^- + \bar{\nu}_e \quad \longleftrightarrow $$



(agreement with experiments) first example of E.F.T.

The extreme case is the β-decay of the free neutron :

$$ n \longrightarrow p + e^- + \bar{\nu}_e $$

Inside a nucleus the neutron is stable because of the $Q_{value} < 0$. A free neutron instead is able to decay because $Q \simeq 1\,MeV > 0$

Why $n \longrightarrow p\pi^-$ or $n \longrightarrow \pi^+\pi^-$ or $n \longrightarrow e^+e^-$ cannot happen? Because of the Baryon number conservation. We are assuming that W.I. conserve the Baryon number (but we have proof only at low energies)

In cosmology there is a theory called Baryogenesis. It tries to describe the process hypothesized to have taken place during the early universe to produce Bayonic asymmetry in the observed universe. In 1967 Sakharov proposed a set of 3 necessary conditions that a baryon-generating interaction must satisfy to produce matter and antimatter at different rates

1) There must be a C/CP violation, and for that we need weak interactions (since EM and STRONG conserve C/CP) $\longrightarrow$ necessary in order to not counterbalance the interactions which produce the excess of baryos with interactions which produce more anti-baryons than baryons (C violation) and in order to not produce an equal number of left-handed and right handed baryons (CP violation)

2) There must be a violation of baryon number : not now but at the beginning of time, at high energies a violation must've occurred $\longrightarrow$ necessary to produce an excess of baryons over anti-baryons

3) Interactions out of thermal equilibrium $\longrightarrow$ necessary because otherwise CPT symmetry would compensate the processes which are increasing and decreasing the Baryon number

So in the following we will assume that weak interactions conserve the baryon number

SYMMETRIES and VIOLATIONS

	P	C	CP	T	Baryon #	Lepton #	Flavor ($\Delta S, \Delta C, \Delta b$)
QED	✓	✓	✓	✓	✓	✓	✓
STRONG	✓	✓	✓	✓	✓	✓	✓
WEAK	✗	✗	✗	✗	✓	✓	✗ ($\Delta S=1$; $\Delta C=1$; $\Delta b=1$)

From Wu experiment

From Goldhaber experiment

From Cronin-Fitch experiment @ SLAC

Since CPT is a universal symmetry and CP is violated → also T has to be violated

e.g. $\Lambda(uds) \to p\pi^-$ $\Delta S=1$

So there are <u>3 main experimental signatures of weak interactions</u>:

1) <u>Production of neutrinos</u>

2) <u>Long life time</u>: in general a long life time could come both from a suppression of the matrix element, due to weak int. and from a small phase space, due to a small Q-value. If the phase space is large but we still have a long life-time → weak interaction

3) <u>Flavor violation</u> (with s, c, or b quarks)

NEUTRINO FLAVORS

We have to answer to 2 questions:

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$$

1) How do we know that $\nu_\mu \neq \nu_e$ ($\neq \nu_\tau$) ?

2) How do we know that $\nu \neq \bar{\nu}$?

- After the β-decay $n \to p + e^- + \bar{\nu}_e$ was known a piece of proof came by the Reines-Cowan experiment in 1956. They put a detector near a reactor and looked for processes like:

$$\bar{\nu}_e + p \longrightarrow n + e^+ \quad \text{(many counts)}$$

$$\bar{\nu}_\mu + p \longrightarrow n + \mu^+ \quad \text{(no counts)}$$

They found a # of events compatible with the expected value confirming the existence of $\bar{\nu}_e$. In the other process they found no events and this is due by the fact that the theshold is much higher respect to the 1st process (where $E_{th} \simeq 2$ MeV). Although they didn't find events because ν didn't have enough energy, the idea to prove that there are two different type of ν is the correct one.

- In 1962 Lederman, Schwarz and Steinberg proved that there are 2 different kind of ν

They used the process:

$$p + Be \longrightarrow \pi^+ + X$$

to produce π^+ and send them against a lot of steel: in this way the pions make an hadronic shower and some of them decay into muons

$$\pi^+ \longrightarrow \mu^+ + \nu_\mu$$

then they looked for events with these 2 different final states and see what happens:

$$\nu_\mu + X \longrightarrow \mu^- + Y$$

$$\nu_\mu + X \longrightarrow e^- + Y$$

The apparatus was the following:

$3.5 \cdot 10^{17}$ protons of 15 MeV on a target of Berillium

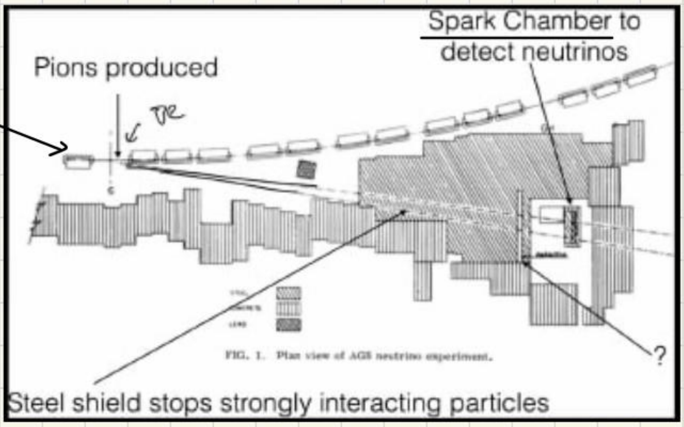


FIG. 1. Plan view of AGS neutrino experiment.

Invented by Conversi: gas between 2 surfaces, when the particle goes through it it produces a spark

They sent $\sim 10^{17}$ protons on the target and observed 34 single μ tracks and 6 electron showers from the spark chambers:

electrons interact more easily (Brems + e.m. shower)

(we know they are e^- and not e^+ from the curvature)

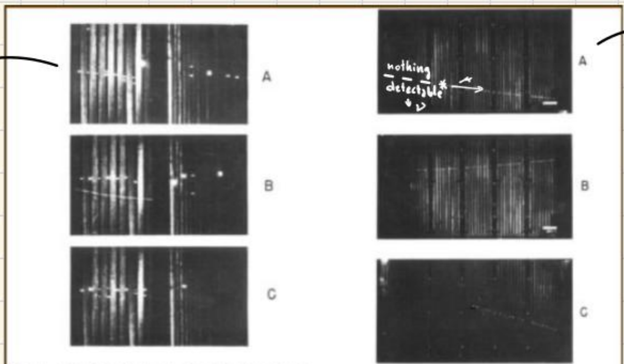


FIG. 8. 400-MeV electrons from the Cosmotron.

FIG. 5. Single muon events. (A) $p_\mu > 540$ MeV and δ ray indicating direction of motion (neutrino beam incident from left); (B) $p_\mu > 700$ MeV/c; (C) $p_\mu > 440$ with δ ray.

μ interacts less and they leave straight lines

If $\nu_\mu \neq \nu_e$ theoretically we shouldn't see any electrons but they saw those electrons for experimental reasons (background). What they concluded is that there are 2 different families: ν_μ produces only μ^- $\longrightarrow$ $\nu_\mu \neq \nu_e$ (Nobel prize)

The same idea of using a neutrino beam and then looking at what comes out was later used for the τ neutrino ν_τ in 1998 at the Donut experiment at Fermilab.

$$\longrightarrow \boxed{\begin{array}{ccc} e^- & \mu^- & \tau^- \\ \nu_e & \nu_\mu & \nu_\tau \\ \\ L_e = +1 & L_\mu = +1 & L_\tau = +1 \end{array}}$$

Phenomenology of weak interaction

Semileptonic decays:
$$n \longrightarrow p\, e^-\, \bar{\nu}_e$$
$$\pi^+ \longrightarrow \mu^+\, \nu_\mu$$

Leptonic decays:
$$\pi^+ \longrightarrow \mu^+\, \nu_\mu$$
$$\mu^+ \longrightarrow \bar{\nu}_\mu\, e^+\, \nu_e$$

Hadronic decays:
$$\Lambda \longrightarrow p\,\pi^- \qquad \Delta S = 1 \qquad (\Delta b = 1,\ \Delta C = 1)$$
$$K^+ \longrightarrow \pi^+\,\pi^0$$

In order to have a theory of Weak interactions it required ~10 years of experiments and theories. The first 2 experiments important to understand this interaction are:

1) <u>Wu experiment</u> : parity violation

2) <u>Goldhaber experiment</u> : there are only left handed neutrinos $\overset{\Leftarrow \vec{\sigma}}{\underset{\vec{p}}{\longrightarrow}}$ $h = -1$

These results tell us that helicity → the handedness and chirality are important in weak interactions

Recap: what is helicity:

For any particle the helicity is defined as $h = \dfrac{\vec{p}\cdot\vec{s}}{|\vec{p}|\cdot|\vec{s}|}$ $\begin{cases} \text{not Lorentz invariant (massive case)} \\ \text{Lorentz invariant (massless case)} \end{cases}$

In the high-energy limit $E \sim p$, $\beta = \frac{p}{E} \sim 1$ helicity is a conserved quantity also for massive particles (e.g. e^- or ν, due to their light mass, are good candidates).

All these considerations let the fact that in 1960's Lee and Yang proposed the so called Chiral theory for weak interactions : weak interactions depends on chirality

We're dealing with fermions and for them we have the Dirac theory for fermions. We denote with ψ the spinor field of a fermion. A spinor field can be splitted in a so called left-handed spinor and in a right-handed spinor:

$$\psi = \boxed{\psi^{LH}} + \boxed{\psi^{RH}}$$

we can do this with the help of the chiral operator

$$\boxed{\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}} \quad \text{with} \quad \boxed{\gamma^0 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \quad \text{Dirac matricies}}$$

$$\boxed{\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{Pauli matricies}}$$

thanks to γ^5, in fact, we can define the chiral projectors:

$$\boxed{P_L = \frac{1}{2}(1 - \gamma^5)} \qquad \boxed{P_R = \frac{1}{2}(1 + \gamma^5)}$$

In this way the left handed and the right handed part of the spinor are given by:

$$\boxed{\psi^{LH} = P_L\psi = \frac{1}{2}(1-\gamma_5)\psi} \qquad \boxed{\psi^{RH} = P_R\psi = \frac{1}{2}(1+\gamma_5)\psi}$$

ψ^{LH} and ψ^{RH} are eigenstates of the chiral operator γ_5 respectively with eigenvalues -1 and $+1$. Indeed:

$$\gamma^5\psi^{LH} = \frac{1}{2}(\gamma^5 - (\gamma^5)^2)\psi = \frac{1}{2}(1-\gamma^5)\psi = -\psi^{LH}$$

$$\gamma^5\psi^{RH} = \frac{1}{2}(\gamma^5 + (\gamma^5)^2)\psi = \frac{1}{2}(1+\gamma^5)\psi = +\psi^{RH}$$

that is called chirality.

What is the relation between helicity and chirality?

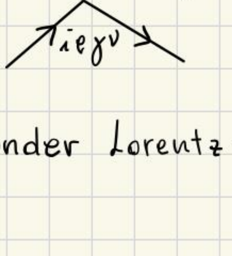
Experimentally we don't know what the chirality can be since it is no made up of observables. The helicity instead is something we can actually measure since it is made up by spin and momentum. We can only extrapolate the chirality on the base of our helicity measurements.

For a massive particle (or antiparticle) with mass $m \rightarrow E = \sqrt{p^2 + m^2}$ if $E \gg m \longrightarrow \psi \simeq \psi^{LH}$ (or $\psi \simeq \psi^{RH}$) The probability of having $\psi = \psi^{LH}$ (or $\psi = \psi^{RH}$) is $1-\beta$: the more $\beta \sim 1$ the more left handedness and right handedness becomes a property of the particle (antiparticle). This means that in the massless (ultra relativistic) limit chirality is a conserved quantity

Helicity depends on momentum and therefore in general it is not Lorentz invariant. However for a massless particle we can't flip the momentum and this means that for a massless particle helicity is Lorentz invariant, it is an intrinsic property of the particle (it does not depend on the frame of reference).
So for a massive particle in general we can swap the sign of momentum with a boost and therefore helicity is not conserved. However in the limit of $E \gg m$ the helicity becomes a good conserved quantity and approximation of chirality with probability β.

How to incorporate chirality in weak interactions?

QED is a current - current interaction:



$$\Rightarrow \mathcal{M} = (\bar{u}\gamma^\mu u)(ie)\frac{-ig_{\mu\nu}}{q^2}(\bar{u}\gamma^\nu u)(ie) = -\frac{e^2}{q^2}j^\mu g_{\mu\nu} j^\nu = -\frac{e^2}{q^2}\underline{j_1 \cdot j_2}$$

where $J^\mu = \overline{\psi}\gamma^\mu\psi$; $\underline{J_1 \cdot J_2} = (J_1^0 J_2^0 - \vec{J}_1 \cdot \vec{J}_2)$

J^μ under Lorentz transformation behaves like a 4 vector. Therefore under parity:

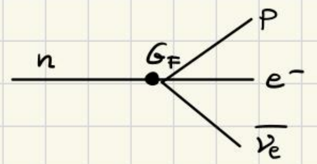
$$\mathbb{P} J^\mu = (\overline{\psi}\gamma^0\psi, -\overline{\psi}\gamma^i\psi)$$

Instead if I apply $\mathbb{P}$ on the matrix element $\mathcal{M}_{QED} = \frac{\alpha}{q^2}(J_1^0 J_2^0 - \vec{J}_1 \cdot \vec{J}_2)$ I get:

$$\boxed{\mathbb{P}\mathcal{M}_{QED} = -\frac{e^2}{q^2}(J_1^0 J_2^0 - (-\vec{J}_1)\cdot(-\vec{J}_2)) = \mathcal{M}_{QED}}$$ $\Rightarrow$ intrinsically QED is invariant under parity (because we have a 4 vector current)

This isn't strange: $\mathcal{M}$ depends on a scalar product of 2 4-vectors (Lorentz invariant). Therefore we can compare results from different frame of reference. (profound meaning) without problems. This feature of QED was a guiding principle to construct the weak interaction theory.

V-A theory



- all the particle touch in one vertex with no mediator

$[G_F] = GeV^{-2}$ (common feature to all contact interaction theories)
(QED is current-current interaction and α is dimensionless)

Fermi was aware of all these facts and he tried then to write the weak interaction as a current-current interaction rather than as a contact interaction:

$$\mathcal{M}_{weak} = G_F \, \underline{J_1} \cdot \underline{J_2}$$

How can I choose the correct which violates parity? I know that:

$$\begin{aligned} \mathbb{P}\vec{v} &= -\vec{v} && \text{(vector)} \\ \mathbb{P}s &= s && \text{(scalar)} \\ \mathbb{P}\vec{A} &= \vec{A} && \text{(pseudo-vector or axial)} \\ \mathbb{P}P_s &= -P_s && \text{(pseudo-scalar)} \end{aligned}$$

- If the current was a 4 vector like $J_w = \psi \gamma^\mu \psi$ there would not be a parity violation!
- If the current is 4-scalar bilinear $\bar{\psi}\psi$ then there is no parity violation!
- In order to build a current that violates parity the solution is to use an axial vector bilinear like $\bar{\psi}\gamma^\mu\gamma^5\psi$ or a tensor bilinear like $\bar{\psi}\sigma_{\mu\nu}\psi$ with $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$
In general we take a combination of them:

$$\boxed{J_{weak} = \bar{\psi}\left(C_V \gamma^\mu + C_A \gamma^5\gamma^\mu + C_T \sigma^{\mu\nu}\right)\psi}$$

C_A, C_V, C_T coefficients to be measured

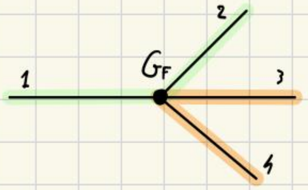
Many years of experiments have been devoted to find these coefficients. The final answer is that

$$\boxed{J_{weak} = \bar{\psi}\gamma^\mu(1-\gamma^5)\psi}$$

V-A current (Modern weak interaction)

So the weak current has both a vector and an axial part and there is a minus in between. The experimental hint to guess this structure comes from the Goldhaber experiment which showed that there are only LH neutrinos: so we could have considered $(1-\gamma^5)$ (that selects the LH part) times the 4 vector current of the neutrino and that is indeed what we have.

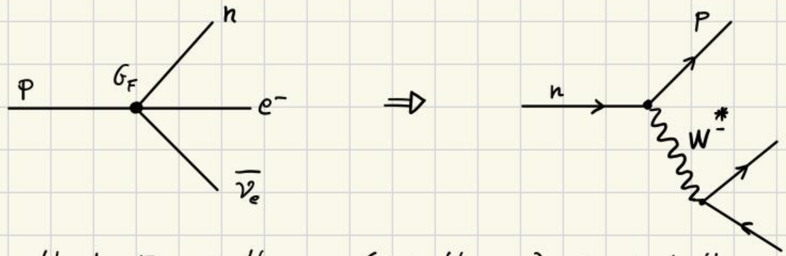
So we rewrote the Fermi theory as a current-current theory (V-A theory): it is a current-current theory but with a factor G_F at the vertex in place of a propagator.



$$\Longrightarrow \mathcal{M} = G_F \, \underline{J}_1 \cdot \underline{J}_2 = G_F \left(\overline{\psi}_1 \gamma^\mu (1-\gamma^5)\psi_2\right)\left(\overline{\psi}_3 \gamma_\mu (1-\gamma^5)\psi_4\right)$$

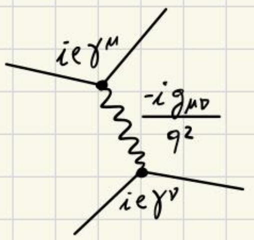
MODERN ELECTRO-WEAK INTERACTION

The idea came to Klein (the same of Klein-Gordon equation) that thought that there could be a massive mediator



The idea is that Fermi theory (V-A theory) is just the low energy limit of the electro-weak theory i.e. increasing the energy we're able to see the mediator.
G_F represents our lack of understanding about what's going on at the fundamental level and now we can relate it to the mass of the mediator.

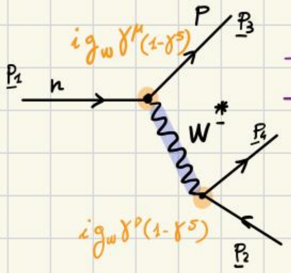
What we have in QED is that:



$$\Longrightarrow \mathcal{M} = \frac{\alpha_{EM}}{q^2}(\dots) \qquad \alpha_{EM} = \frac{e^2}{4\pi}$$

(in every vertex there's a factor $i\sqrt{\alpha}$)

With weak-interaction we can do the same thing:



$$\frac{-i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_w^2}\right)}{q^2 - M_w^2}$$

g_w : weak charge (equivalent to e)
M_w : mediator mass
$\underline{q} = \underline{p}_1 - \underline{p}_3$

Experimentally we see that $M_w \simeq 80$ GeV : we change q finding resonance peaks which correspond to the case where $q \sim M_w$ i.e. where $\mathcal{M} \to \infty$.

In the β neutron decay what happens is that:

$$n \longrightarrow p + e^- + \overline{\nu}_e \qquad \to Q_{value} = m_n - m_p - m_e \simeq 1 \text{ MeV}$$

Therefore at the best we have:

$$q = 1 \text{ MeV} \longrightarrow q^2 \ll M_w^2 \Longrightarrow \boxed{\mathcal{M} = \frac{-i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_w^2}\right)}{q^2 - M_w^2} \simeq \frac{-i\, g_{\mu\nu}}{M_w^2}}$$

So the interaction collapses into a point: we can associate this it to G_F:

$$G_F \sim \frac{g_w^2}{M_w^2} \xrightarrow{\text{doing calculation correctly}} \boxed{G_F = \frac{\sqrt{2}}{8}\frac{g_w^2}{M_w^2}}$$

As we said before g_W is the weak charge and it is the weak equivalent of the electric charge e. By analogy with QED we can say that:

$$\alpha_{EM} = \frac{e^2}{4\pi} \longrightarrow \boxed{\alpha_W = \frac{g_W^2}{4\pi}}$$

Both the Fermi constant and the mass of the W can be measured experimentally and this gives us the access to the weak charge:

$$M_W \simeq 80 \text{ GeV} \; ; \; G_F \simeq 1.16 \cdot 10^{-5} \text{ GeV}^{-2} \longrightarrow \boxed{g_W = 0.653}$$

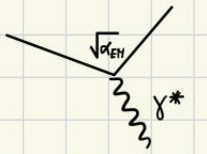
this means that:

$$\boxed{\alpha_W = \frac{g_W^2}{4\pi} = \frac{1}{29.5} > \alpha_{EM} = \frac{e^2}{4\pi} = \frac{1}{137}}$$

$$\alpha_{strong} = \frac{g_{strong}^2}{4\pi} \simeq 0.1$$

So the weak interaction is not intrinsically weak $(\alpha_W > \alpha_{EM})$ but the difference is in the propagator which suppresses the weak interaction matrix element:

E.M.



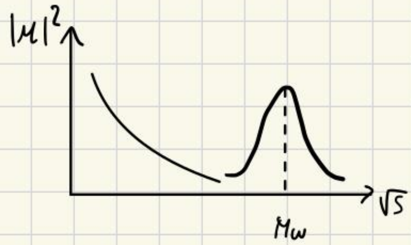
$$\sim \frac{1}{q^2} \Rightarrow \mathcal{M}_{EM} \propto \frac{\alpha_{EM}}{q^2} \xrightarrow{q^2 \ll M_W^2} \frac{\alpha_{EM}}{q^2}$$

$$\Rightarrow \frac{\mathcal{M}_{WEAK}}{\mathcal{M}_{EM}} \sim \frac{q^2}{M_W^2}\frac{\alpha_W}{\alpha_{EM}} \sim 10^{-10}$$

WEAK

$$\sim \frac{1}{q^2 - M_W^2} \Rightarrow \mathcal{M}_{WEAK} \propto \frac{\alpha_W}{q^2 - M_W^2} \longrightarrow \frac{\alpha_W}{M_W^2}$$

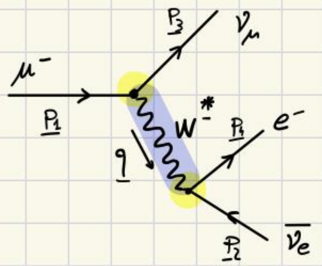
All the processes that we've seen happened at the scale of no more 100 MeV so the $M_W \sim 80$ GeV is completely out of scale and this means we are always in the low energy limit. where we cannot see the propagator and where the Fermi-V-A theory is completely fine approximation.
To actually distinguish the Fermi theory from the propagator theory we have to build a machine capable of giving a momentum transfer of the order of the M_W wher $\mathcal{M}_{WEAK}$ has a pole. i.e. resonance



Applications of weak interactions

① μ- DECAY :

$$\boxed{\mu^- \longrightarrow e^- \bar{\nu}_e \nu_\mu} \Longrightarrow$$



$$\underline{p_1} = \underline{p_3} + \underline{q} \longrightarrow \underline{q} = \underline{p_1} - \underline{p_3}$$

$$Q_{value} = m_\mu - m_{e^-} \sim 105 \text{ MeV}$$

The matrix element is:

$$\mathcal{M} = \frac{g_w^2}{q^2 - M_w^2}\left[\bar{u}(p_3)\,\gamma^\mu(1-\gamma^5)\,u(p_1)\right] \times \left[\bar{u}(p_4)\,\gamma_\mu(1-\gamma^5)\,v(p_2)\right]$$

the goal is to compute :

$$\Gamma(\mu^- \longrightarrow e^- \nu_\mu \bar{\nu}_e) \propto |\mathcal{M}|^2 \times (\text{phase space})$$

- we need to compute is $|\mathcal{M}|^2$, or better $\langle|\mathcal{M}|^2\rangle$:

$$\langle|\mathcal{M}|^2\rangle = \sum_{\substack{\text{spins in}\\ \text{final state}}} \cdot \sum_{\substack{\text{spin in}\\ \text{initial}\\ \text{state}}} \mathcal{M}^2$$

in the final state there are many possible spin configurations that we don't look → we sum over them.

in the initial state we don't use a polarized beam of μ so we sum also over the all spin configurations in the initial state.

<u>Initial state</u> : $-\mu^-$ with 2 spin states (since $m_\mu \neq 0$) or

<u>Final states</u> :
- ν_μ with 1 spin state
- e^- with 2 spin states
- $\bar{\nu}_e$ with 1 spin state

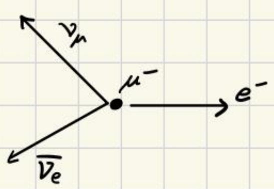
From these sums we get a trace of γ matricies. ⟶ easier calculations.

- Then we see that the scale of the energy (see Q_{value}) is such that at the best $q \simeq 105$ MeV and therefore $q^2 \ll M_w^2$

$$\Longrightarrow \Gamma \propto \left(\frac{g_w^2}{M_w^2}\right)^2 (\dots) \times (\text{phase space}) \longrightarrow \Gamma \propto G_F^2 \times (\text{phase space})$$

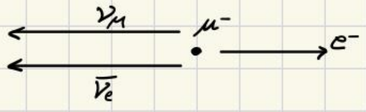
trace of γ matricies

Let's now put ourself in the μ rest frame to do calculations. A general configuration is :



There are <u>2 extreme configurations</u> :

1)



$$E_e = E_e^{max} = \frac{1}{2} m_\mu \Longrightarrow p_e = p_e^{max} \simeq \frac{1}{2} m_\mu$$

2) $\overset{\nu_\mu}{\longleftarrow} \;\; \overset{\uparrow e^-}{\underset{\mu^-}{\bullet}} \longrightarrow \bar{\nu}_e$ $\qquad E_e = E_e^{min} \simeq m_e \;\Rightarrow\; P_e = P_e^{min} \simeq 0$

we care about the energy of the electron because is the only thing we can see.
The phase space takes into account all the possible configurations that we have. It has 3 integrals (3 body decay):

$$\rho_{\substack{phase\\space}} = \frac{d^3P_{\nu_e}}{(2\pi)^3}\frac{d^3P_{\nu_\mu}}{(2\pi)^3}\frac{d^3P_{e^-}}{(2\pi)^3}\,\delta^4(P_\mu - P_e - P_{\nu_e} - P_{\nu_\mu})$$

From a dimensional point of view

$$[\rho] = E^5 \text{ in fact } \Gamma \sim G_F^2 \rho \;;\; [\Gamma] = E \;;\; [G_F] = E^{-2} \longrightarrow [\rho] = E^5$$

So the more energy we have the larger the phase space will be. In our case all the Kinematics depends on the mass of the μ. Larger is m_μ larger is the energy transfered to the decay's products m_μ is the energy scale of the process $(Q \sim m_\mu)$. Therefore we could say that:

$$\Gamma \propto G_F^2 m_\mu^5 \quad \text{(from dimensional analysis)}$$

Computing the full integral:

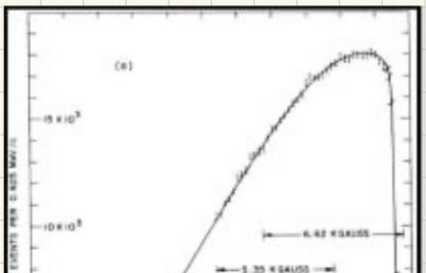
$$\boxed{\Gamma = \frac{1}{192\pi^3} G_F^2 m_\mu^5} \quad \text{(Total width)}$$

Therefore this means that:

$$\boxed{\tau_\mu = \frac{1}{\Gamma_\mu} \propto m_\mu^{-5}}$$

so the heavier is the decaying particle the faster it decays. What we've seen is the total width, but we could also consider the differential width and compare it with measurements:

\# decays as function of E_e



$\sim \frac{1}{2} m_\mu$

- Spectacular agreement between theory and experiment
- the form of the function is the one we expect: it goes at zero at some maximum energy ($E_{max} \sim \frac{1}{2} m_\mu$) and it goes to zero at zero energy ($E_{min} \sim m_e \sim 0$)

Moreover counting the number of events (without looking at the whole spectrum) we can measure the lifetime τ. Then with a spectrometer we can measure m_μ. This allow us to have a precise measurement of G_F:

$$\frac{1}{\tau_{meas}} = \frac{1}{192\pi^3} G_F^2 m_{\mu_{meas}}^5 \longrightarrow G_F = \sqrt{\frac{192\pi^3}{\tau_{meas} \cdot m_{\mu_{meas}}^5}}$$

So we are able to measure the parameters of the theory. Moreover if we find a way to measure independently g_w we could find a way to measure M_W without actually producing it (this was done obtaining $M_W \sim 80$ GeV).

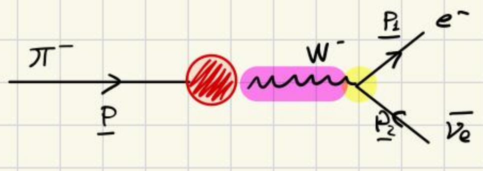
② π - DECAY

We have 2 possible processes:

$$\pi^- \longrightarrow e^- \overline{\nu_e} \qquad Q_e = m_\pi - m_e \simeq 135 \text{ MeV}$$

$$\pi^- \longrightarrow \mu^- \overline{\nu_\mu} \qquad Q_\mu = m_\pi - m_\mu \simeq 29 \text{ MeV}$$

If we were to take a bet we'd say that the favored decay is the one into electrons because the phase space (related to Qvalue) is larger. However in 1958 @ CERN Fidecaro measured BR ($\pi^- \to e^- \overline{\nu}_e$) $\neq 0$ but very very small. As we'll see this is due to the weak interaction. Let's take the diagram for the e⁻ channel:



the matrix element is

$$\mathcal{M} = \frac{g_w^2}{q^2 - M_W^2} \left(\overline{u}(P_3)\, \gamma^\mu (1-\gamma^5)\, v(P_2) \right) \times F_\mu$$

this the vertex that we don't Know
In order to have a scalar matrix element
it has to be a 4-vector

(N.B. Also in this case $|q| \sim m_\pi \ll M_W \longrightarrow \frac{g_w^2}{q^2 - M_W^2} \sim \frac{g_w^2}{M_W^2} = G_F$)

Let's put ourself in the reference frame of π and build F_μ out of the quantities that we have : we can either use the spin of the π or its 4-momentum. Since the π's spin is equal to Zero we can only use its 4-momentum.

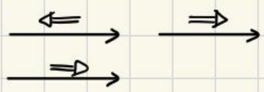
$$\longrightarrow \boxed{F_\mu = f_\pi \cdot P_\mu}$$

f_π is the pion-decay constant.

Then we can go on as before and compute the average matrix element $\langle |\mathcal{M}|^2 \rangle$ taking into account that:

Initial state : - π^- with 1 spin state

Final state : - e^- with 2 spin states
- $\overline{\nu_e}$ with 1 spin state



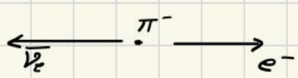
$$\longrightarrow \boxed{\langle |\mathcal{M}|^2 \rangle = \left(\frac{g_W}{2M_W} \right)^4 f_\pi^2\, m_e^2\, (m_\pi^2 - m_e^2)}$$

The decay width is then given by

$$\Gamma \propto \langle |\mathcal{M}|^2 \rangle \times (\text{phase space})$$

Integrating we find that:

$$\boxed{\Gamma = \frac{1}{8\pi} \frac{|\vec{P}_2|}{m_\pi^2} \langle |\mathcal{M}|^2 \rangle \qquad \text{with } |\vec{P}_2| = \frac{1}{2m_\pi}(m_\pi^2 - m_e^2)}$$

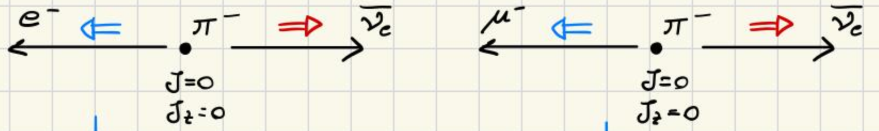


the only thing that we do not Know is the value of f_π.

However if we look at the ratio between the 2 decay channels (and this is what we are interested in) it disappears. (also from an exp. point of view we can cancel systematic errors)

$$\boxed{\frac{\Gamma(\pi^- \to e^-\bar{\nu}_e)}{\Gamma(\pi^- \to \mu^-\bar{\nu}_\mu)} = \frac{m_e^2}{m_\mu^2}\frac{(m_\pi^2 - m_e^2)^2}{(m_\pi^2 - m_\mu^2)^2} \approx 1.283 \cdot 10^{-4}} \longrightarrow e^- \text{ channel is suppressed}$$

<u>Why is it so suppressed</u>?



↓ We need a RH e^- in order to counter balance.	↓ We need a RH μ^- in order to counter balance.
↓ The probability of producing a RH. e^- is prop. to $(1-\beta)$	↓ The probability of producing a RH. μ^- is prop. to $(1-\beta)$
Since $\beta_{e^-} = \frac{p_e}{E_e} \simeq 1 - 2.6 \cdot 10^{-5} \sim 1$	Since $\beta_\mu = \frac{p_\mu}{E_\mu} \simeq 0.38$
$\Longrightarrow$ prob (RH e^-) ~ 0 %	$\Longrightarrow$ prob (RH e^-) ~ 62 %
⇓ SUPPRESSED!	

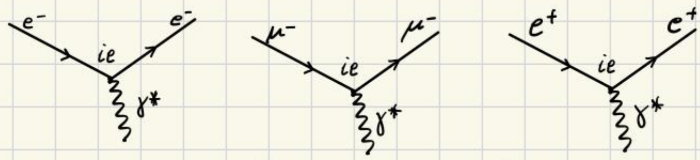
In a universe with $m_e = 0$ the lifetime of the π would be longer because the decay with the e^- would be completely forbidden.

In a universe with $m_e = m_\mu = 0$ the π would become stable because the decay could not happen also because there are no lighter hadrons to which it could decay strongly.

Lepton Flavor universality of weak interactions

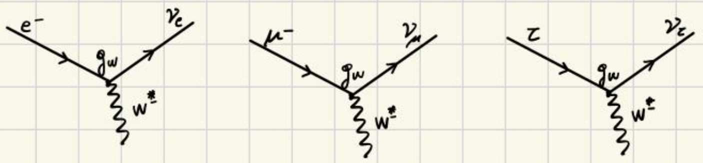
<u>How to prove that</u>: $\boxed{g_W^e \equiv g_W^\mu \equiv g_W^\tau}$

Let's look back at Q.E.D.



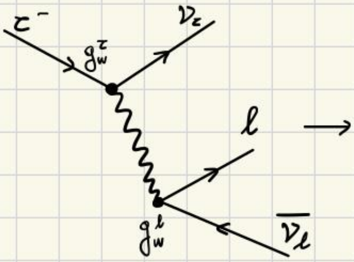
How do we know that $g_{EM} = e$ in the vertex is the same? We know it because we can see experimentally that $|Q_{e^-}| \equiv |Q_{\mu^-}| = |Q_{e^+}|$ (for example applying a magnetic field and extrapolate Q from p=QRB).

In the same way we want tho know whether the g_W in the vertex is the same for every particle or not.



To answer this question we have to find a process that can access to all the coupling constants. We know that such a process is the τ decay (it can decays both to e^- or μ^-).

$m_\tau \simeq 1.78$ GeV
$m_\mu \simeq 105$ MeV
$m_e \simeq 0.5$ MeV



$\longrightarrow \ell = e^-$ or μ^-

The decay width is:

$$\Gamma(\tau^- \to e^- \overline{\nu}_e \nu_\tau) = \frac{(g_w^\tau)^2 (g_w^e)^2}{(m_w)^4} m_\tau^5 \, \rho_{\text{Phase space}}(\tau^- \to e^- \overline{\nu}_e \nu_\tau)$$

$$\Gamma(\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau) = \frac{(g_w^\tau)^2 (g_w^\mu)^2}{(m_w)^4} m_\tau^5 \, \rho_{\text{Phase space}}(\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau)$$

$$\xrightarrow{\Gamma_{tot} = \sum_f \Gamma(\tau^- \to f)}$$

$$BR(\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau) = \frac{\Gamma(\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau)}{\Gamma_{tot}}$$

$$BR(\tau^- \to e^- \overline{\nu}_e \nu_\tau) = \frac{\Gamma(\tau^- \to e^- \overline{\nu}_e \nu_\tau)}{\Gamma_{tot}}$$

Experimentally we can measure the BR as: $BR(\tau^- \to f) = \frac{\#\text{ events } \tau^- \to f}{\#\text{ total } \tau^- \text{ produced}}$

We can compute the ratio between $\Gamma(\tau^- \to \mu^-)$ and $\Gamma(\tau^- \to e^-)$ as:

$$\frac{\Gamma(\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau)}{\Gamma(\tau^- \to e^- \overline{\nu}_e \nu_\tau)} = \frac{BR(\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau)}{BR(\tau^- \to e^- \overline{\nu}_e \nu_\tau)} \frac{\cancel{\Gamma_{tot}}}{\cancel{\Gamma_{tot}}}$$

computed experimentally

and compare this result with:

$$\frac{\Gamma(\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau)}{\Gamma(\tau^- \to e^- \overline{\nu}_e \nu_\tau)} = \frac{(g_w^\mu)^2}{(g_w^e)^2} \frac{\rho(\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau)}{\rho(\tau^- \to e^- \overline{\nu}_e \nu_\tau)}$$

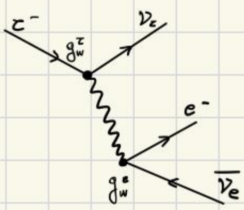
computed analytically

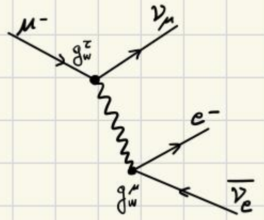
$$\longrightarrow \frac{(g_w^\mu)^2}{(g_w^e)^2} = \frac{BR(\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau)}{BR(\tau^- \to e^- \overline{\nu}_e \nu_\tau)} \frac{\rho(\tau^- \to \mu^- \overline{\nu}_\mu \nu_\tau)}{\rho(\tau^- \to e^- \overline{\nu}_e \nu_\tau)} \simeq 0.976$$

$$\longrightarrow \boxed{\frac{g_w^\mu}{g_w^e} \simeq 1.001 \pm 0.002} \Longrightarrow \boxed{g_w^e \equiv g_w^\mu} \longrightarrow \text{(proved the } e \leftrightarrow \mu \text{ universality)}$$

Now we have to prove it also for g_w^τ. Proving that we'll also obtain the proof that τ is a lepton because we'll have the experimental evidence that τ behaves like a lepton under weak interactions. Let's look at the B.R. of the 2 decays:

$\tau^- \to e^- \overline{\nu}_e \nu_\tau$ $\qquad$ $\mu^- \to e^- \overline{\nu}_e \nu_\mu$ $\longrightarrow$ why whe choose this? Because it is very frequent $BR(\mu^- \to e^- \overline{\nu}_e \nu_\mu) \sim 100\%$





taking the ratio between the 2 B.R. we'll find the ratio $\frac{g_\tau}{g_\mu}$:

$$\frac{\Gamma(\mu^- \to e^- \overline{\nu}_e \nu_\mu)}{\Gamma(\tau^- \to e^- \overline{\nu}_e \nu_\tau)} = \frac{BR(\mu^- \to e^- \overline{\nu}_e \nu_\mu)}{BR(\tau^- \to e^- \overline{\nu}_e \nu_\tau)} \frac{\Gamma_{tot}^\mu}{\Gamma_{tot}^\tau} = \frac{BR(\mu^- \to e^- \overline{\nu}_e \nu_\mu)}{BR(\tau^- \to e^- \overline{\nu}_e \nu_\tau)} \frac{\tau_\tau}{\tau_\mu}$$

experimentally measured

(n.b. $\lambda = \beta\gamma c\tau$)

that we can compare with:

$$\frac{\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu)}{\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} = \frac{(g_w^\mu)^2}{(g_w^\tau)^2} \frac{m_\mu^5}{m_\tau^5} \frac{\rho(\mu^- \to e^- \bar{\nu}_e \nu_\mu)}{\rho(\tau^- \to e^- \bar{\nu}_e \nu_\tau)}$$

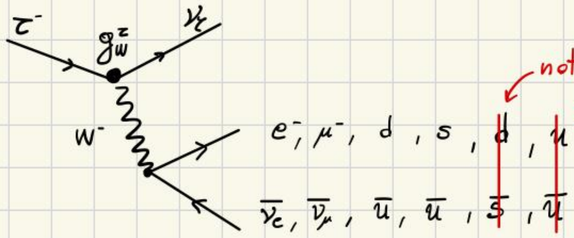
computed analytically

$$\longrightarrow \boxed{\frac{g_w^\mu}{g_w^\tau} \simeq 1.001 \pm 0.003} \longrightarrow \boxed{g_w^\tau \equiv g_w^\mu} \longrightarrow \tau \leftrightarrow \mu \text{ universality proved}$$

$$\longrightarrow \boxed{g_w^\mu \equiv g_w^\tau \equiv g_w^e \equiv g_w}$$

τ decay in details

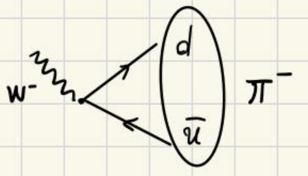
τ can in principle decays into electrons, muons and also hadrons

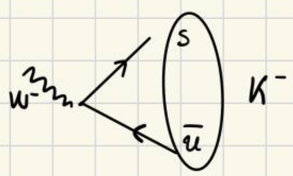


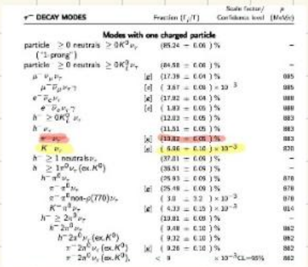
not allowed kinematically $Q<0$

not allowed for charge cons.

Let's have a look at the hadronic decays:

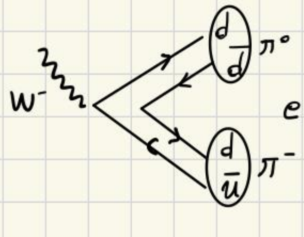




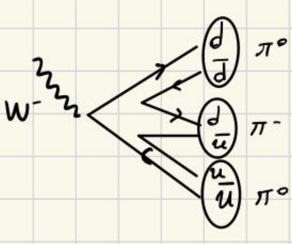


τ^- DECAY MODES		Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
Modes with one charged particle				
particle ≥ 0 neutrals $\geq 0K^0\nu_\tau$ ("1-prong")		(85.24 ± 0.06) %		–
particle ≥ 0 neutrals $\geq 0K^0_L\nu_\tau$		(84.58 ± 0.06) %		–
$\mu^-\bar{\nu}_\mu\nu_\tau$	[g]	(17.39 ± 0.04) %		885
$\mu^-\bar{\nu}_\mu\nu_\tau\gamma$	[e]	$(3.67 \pm 0.08) \times 10^{-3}$		885
$e^-\bar{\nu}_e\nu_\tau$	[g]	(17.82 ± 0.04) %		888
$e^-\bar{\nu}_e\nu_\tau\gamma$	[e]	(1.83 ± 0.05) %		888
$h^- \geq 0K^0_L\nu_\tau$		(12.03 ± 0.05) %		883
$h^-\nu_\tau$		(11.51 ± 0.05) %		883
$\pi^-\nu_\tau$	[g]	(10.82 ± 0.05) %		883
$K^-\nu_\tau$	[g]	$(6.96 \pm 0.10) \times 10^{-3}$		820
$h^- \geq 1$ neutrals ν_τ		(37.01 ± 0.09) %		–
$h^- \geq 1\pi^0\nu_\tau$ (ex. K^0)		(36.51 ± 0.09) %		–
$h^-\pi^0\nu_\tau$		(25.93 ± 0.09) %		878
$\pi^-\pi^0\nu_\tau$	[g]	(25.49 ± 0.09) %		878
$\pi^-\pi^0$ non-$\rho(770)\nu_\tau$		$(3.0 \pm 3.2) \times 10^{-3}$		878
$K^-\pi^0\nu_\tau$	[g]	$(4.33 \pm 0.15) \times 10^{-3}$		814
$h^- \geq 2\pi^0\nu_\tau$		(10.81 ± 0.09) %		–
$h^-2\pi^0\nu_\tau$		(9.48 ± 0.10) %		862
$h^-2\pi^0\nu_\tau$ (ex. K^0)		(9.32 ± 0.10) %		862
$\pi^-2\pi^0\nu_\tau$ (ex. K^0)	[g]	(9.26 ± 0.10) %		862
$\pi^-2\pi^0\nu_\tau$ (ex. K^0),		< 9	$\times 10^{-3}$ CL=95%	862

We find experimentally that $BR(\tau^- \to K^- \nu_\tau) \simeq 10^{-3} \times BR(\tau^- \to \pi^- \nu_\tau)$. The reason cannot be in the phase space but in the Cabibbo angle which represents the fact that universality is broken for quarks. The decay with just 1 pion is the one with larger phase space because the final state is the lightest but playing with QCD we can also consider:



e.g. $\tau^- \longrightarrow \nu_\tau + \pi^- + \pi^0$



e.g. $\tau^- \longrightarrow \nu_\tau \pi^0 \pi^0 \pi^-$

So the main decay channels seems to be e^-, μ^-, π^-. Let's compare the 3 different decays: in terms of decay widths $\Gamma_{\tau\to\mu}, \Gamma_{\tau\to e}, \Gamma_{\tau\to\bar{u}d}$.
Experimentally we already saw that $\Gamma_{\tau\to\mu^-} \simeq \Gamma_{\tau\to e^-}$ in fact from PDG $\Gamma_{\tau\to e^-} \simeq \Gamma_{\tau\to\mu^-} \simeq 17\%$.
Instead for the hadronic decay we find that $\Gamma_{\tau\to\bar{u}d} \simeq 37\% \simeq 3 \times \Gamma_{\tau^-\to e^-}$. This is due to the fact that quarks have color so we have the diagram with $\bar{u}$ and d 3 times and this implies a B.R. 3 times larger. Therefore:

$$\Gamma_{tot} \simeq \Gamma_{\tau\to e^-} + \Gamma_{\tau\to\mu} + \Gamma_{\bar{u}d} \simeq 5\,\Gamma_{\tau\to e^-}$$

$$\longrightarrow \Gamma_{\tau\to e^-} \simeq \frac{1}{5}\Gamma_{tot} = \frac{1}{5}\frac{1}{\tau_\tau}$$

Taking the ratio :

because universality

$$\frac{\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)}{\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu)} = \frac{m_\tau^5}{m_\mu^5}\frac{\cancel{G_F^2}}{\cancel{G_F^2}} \cdot \underbrace{\frac{\rho(\tau^- \to e^- \bar{\nu}_e \nu_\tau)}{\rho(\mu^- \to e^- \bar{\nu}_e \nu_\mu)}}_{\simeq 1 \ (m_\tau, m_\mu \gg m_e)} \longrightarrow \Gamma(\tau^- \to e^-) \simeq \Gamma(\mu^- \to e^-)\frac{m_\tau^5}{m_\mu^5}$$

Therefore:

$$\frac{1}{5}\frac{1}{\tau_\tau} \simeq \Gamma(\mu^- \to e^-)\frac{m_e^5}{m_\mu^5} \simeq \frac{1}{\tau_\mu}\frac{m_e^5}{m_\mu^5} \longrightarrow \boxed{\tau_\tau \simeq 5 \cdot \tau_\mu \cdot \left(\frac{m_\mu}{m_\tau}\right)^5}$$

Therefore using that $\tau_\mu \simeq 2.2 \cdot 10^{-6}\,s$ and $\frac{m_\mu}{m_\tau} \simeq \frac{106}{1780} \simeq 0.06$

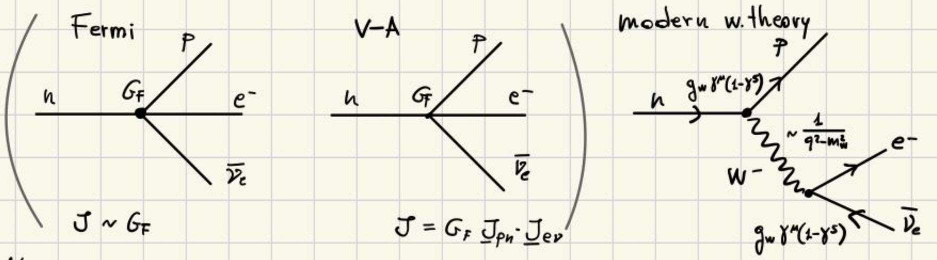
$$\rightarrow \boxed{\tau_\tau^{exp} \simeq 2.9 \cdot 10^{-13}\,s} \quad \text{(good agreement with } \tau_\tau^{PDG} = 3.1 \cdot 10^{-3}\,s\text{)}$$

The agreement between the 2 results means that the assumption of the lepton universality is correct: basically another proof.

Weak interaction of hadrons

Let's go back to β^- decay : $n \to p e^- \bar{\nu}_e$.

only for remember the history of what we've done ←



We compute the matrix element as:

$$\mathcal{M} \sim g_w^2\, \bar{\psi}_p \gamma^\mu(1-\gamma^5)\psi_n \left(\frac{1}{q^2 - m_w^2}\right) \bar{\psi}_{\bar{\nu}} \gamma^\mu(1-\gamma^5)\psi_e \qquad \text{but } q \simeq m_n \ll m_W$$

$$\simeq g_w^2\, \bar{\psi}_p \gamma^\mu(1-\gamma^5)\psi_n \left(\frac{1}{m_w^2}\right) \bar{\psi}_{\bar{\nu}} \gamma^\mu(1-\gamma^5)\psi_e$$

Consequently the amplitude as : $\Gamma \sim \langle|\mathcal{M}|^2\rangle \cdot \rho_{\text{phase space}}$, where $\langle|\mathcal{M}|^2\rangle$ is the mean of $|\mathcal{M}|^2$ over all the initial and final spin configurations:

<u>initial state</u> $\quad n$: $\overset{\Leftarrow}{\rightarrow} \quad \overset{\Rightarrow}{\rightarrow}$

<u>final state</u> $\quad p$: $\overset{\Leftarrow}{\rightarrow} \quad \overset{\Rightarrow}{\rightarrow}$

$\bar{\nu}_e$: $\overset{\Rightarrow}{\rightarrow}$

e^- : $\overset{\Leftarrow}{\rightarrow} \quad \overset{\Rightarrow}{\rightarrow}$

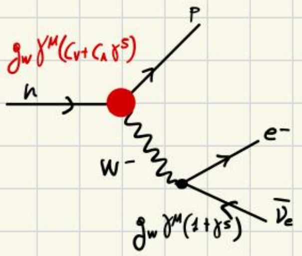
Therefore doing the calcolation we find

$$\Gamma_{theory}(n \to p e^- \bar{\nu}_e) \longrightarrow \boxed{\tau_{theory} = \frac{1}{\Gamma_{theory}} \simeq 1318\,s}$$

However experimentally we have that:

$$\boxed{\tau_{exp} \simeq 878\,s}$$

This discrepancy implies that hadrons behave differently. We should consider the general case with different vector and axial couplings:

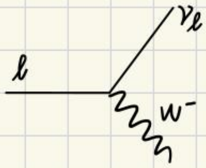


$$\longrightarrow \mathcal{M} \sim \frac{g_w^2}{m_W^2} \overline{\psi}_p \gamma^\mu (C_V + C_A \gamma^5) \psi_n \cdot \overline{\psi}_{\bar{\nu}} \gamma^\mu (1-\gamma^5) \psi_e$$

where C_V and C_A are measured by β^- / β^+ decays.

WEAK INTERACTION IN QUARKS

With leptons we've seen the process:



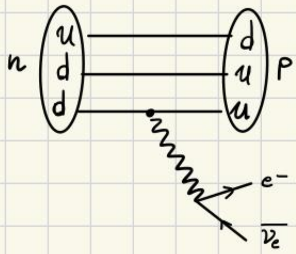
where $\ell \in \{e, \mu, \tau\}$ $\rightarrow$ we say that W^- couples with:

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix}, \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}, \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$$

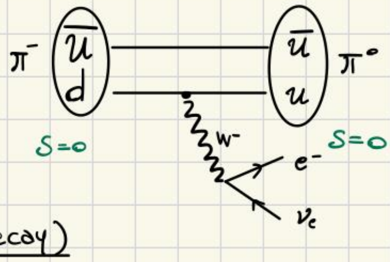
But we haven't seen any *lepton flavor violation* like e^- — ν_μ, W^-. With hadrons we'll see that this is not true. First of all let's have a look to some well known process.

- Neutron $n = (udd)$ β^- decay (semileptonic decay)

There is only one quark which is partecipating in the weak interaction and 2 other quarks that do not interact. These 2 quarks are called spectator quarks

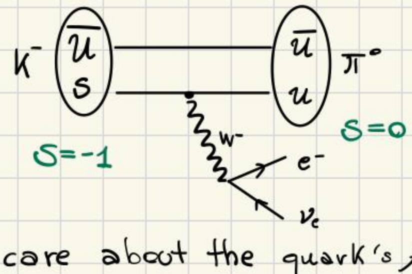


- π^- decay (semileptonic decay)



$\longrightarrow \Delta S = 0$

- K^- decay (semileptonic decay)



$\longrightarrow \Delta S = 1$

So, it seems that W^- does not care about the quark's flavor conservation. In fact as we can see in this last case W^- couples s to u and this produces a strangeness violation $\Delta S = 1$
So we see that both the decays with or without strangeness violation can happen: let's compare the 2 decays in order to understand the difference.

1^{st} evidence of the difference (with mesons π^+ and K^+)

Let's have a look to the leptonic decays of π^+ and K^+. π^+ and K^+ are essentially the same except for $d \leftrightarrow s$ (and $m_\pi \simeq 140$ MeV / $m_K \simeq 500$ MeV)
Seeing the p.d.g. tables we can see that

π^+ DECAY MODES

π^- modes are charge conjugates of the modes below.

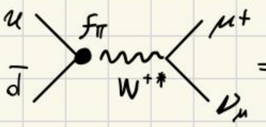
For decay limits to particles which are not established, see the section on Searches for Axions and Other Very Light Bosons.

	Mode		Fraction (Γ_i/Γ)	Confidenc
Γ_1	$\mu^+\nu_\mu$	[a]	(99.98770 ± 0.00004) %	
Γ_2	$\mu^+\nu_\mu\gamma$	[b]	$(2.00 \pm 0.25) \times 10^{-4}$	
Γ_3	$e^+\nu_e$	[a]	$(1.230 \pm 0.004) \times 10^{-4}$	
Γ_4	$e^+\nu_e\gamma$	[b]	$(7.39 \pm 0.05) \times 10^{-7}$	
Γ_5	$e^+\nu_e\pi^0$		$(1.036 \pm 0.006) \times 10^{-8}$	

K^+ DECAY MODES	Fraction (Γ_i/Γ)	Scal Confide
	Leptonic and semileptonic modes	
$e^+\nu_e$	$(1.582 \pm 0.007) \times 10^{-5}$	
$\mu^+\nu_\mu$	(63.56 ± 0.11) %	
$\pi^0 e^+\nu_e$	(5.07 ± 0.04) %	

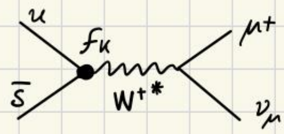
→ this means that there should be a difference between the 2 diagrams.

π^+ :



$$\Rightarrow \boxed{\mathcal{M} \sim f_\pi \frac{g_W^2}{m_W^2} m_\mu (m_\pi^2 - m_\mu^2)} \quad ; \quad \boxed{\Gamma(\pi^+ \to \mu^+\nu_\mu) \sim G_{F_\pi}^2 f_\pi^2 \frac{m_\mu^2}{m_\pi^3} (m_\pi^2 - m_\mu^2)^2}$$

K^+ :



$$\boxed{\mathcal{M} \sim f_K \frac{g_W^2}{m_W^2} m_\mu (m_\pi^2 - m_\mu^2)} \qquad \boxed{\Gamma(K^+ \to \mu^+\nu_\mu) \sim G_{F_K}^2 f_K^2 \frac{m_\mu^2}{m_K^3} (m_K^2 - m_\mu^2)^2}$$

If we take the ratio between the two Γs :

$$\frac{\Gamma(K^+ \longrightarrow \mu^+\nu_\mu)}{\Gamma(\pi^+ \longrightarrow \mu^+\nu_\mu)} = \frac{G_{F_K}^2}{G_{F_\pi}^2} \left(\frac{f_K}{f_\pi}\right)^2 \left(\frac{m_\pi}{m_K}\right)^3 \frac{(m_K^2 - m_\mu^2)^2}{(m_\pi^2 - m_\mu^2)^2}$$

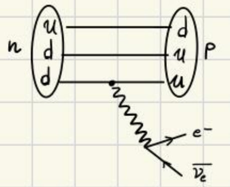
Remember that: $\Gamma(\dots) = BF(\dots)\,\Gamma_{tot} \propto \frac{\#\text{ decays}}{\#\text{ tot decays}}$. So we can measure the ratio between Γs experimentally (see p.d.g tables above) What we get is:

$$\boxed{\left(\frac{G_{F_K}}{G_{F_\pi}}\right)^2 \simeq 0.05}$$

This tells us that weak interaction is less likely to happen if we have $\Delta S = 1$

2^{nd} evidence of the difference (with baryons n, Σ^-)

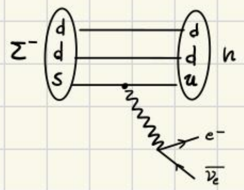
n :



$$\Gamma \propto G_F^2(\Delta S = 0)\, \rho_{\text{phase space}}$$

	Mode		Fraction (Γ_i/Γ)	Confidence level
	n DECAY MODES			
Γ_1	$pe^-\bar{\nu}_e$		100 %	
Γ_2	$pe^-\bar{\nu}_e\gamma$	[a]	$(9.2 \pm 0.7) \times 10^{-3}$	
Γ_3	hydrogen-atom $\bar{\nu}_e$		$< 2.7 \times 10^{-3}$	95%

Σ^-:



$$\Gamma \propto G_F^2(\Delta S = 1)\, \rho_{\text{phase space}}$$

	Mode		Fraction (Γ_i/Γ)	Confidence level
	Σ^- DECAY MODES			
Γ_1	$n\pi^-$		(99.848 ± 0.005) %	
Γ_2	$n\pi^-\gamma$	[a]	$(4.6 \pm 0.6) \times 10^{-4}$	
Γ_3	$ne^-\bar{\nu}_e$		$(1.017 \pm 0.034) \times 10^{-3}$	
Γ_4	$n\mu^-\bar{\nu}_\mu$		$(4.5 \pm 0.4) \times 10^{-4}$	
Γ_5	$\Lambda e^-\bar{\nu}_e$		$(5.73 \pm 0.27) \times 10^{-5}$	
Γ_6	$\Sigma^+ X$		$< 1.2 \times 10^{-4}$	90%

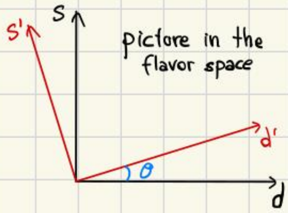
The logic is the same as before and by experimental measurements of masses, decay constants and branching fraction we can obtain in the end the extimate of the ratio between the Fermi constants and what we find is, also in this case :

$$\boxed{\frac{G_F^2(\Delta S = 1)}{G_F^2(\Delta S = 0)} \simeq 0.05}$$

The solution to this problem came by Cabibbo. The idea was the following. In nature we have 3 (eigenstates flavors) u, d, s. (in 1963 they only knew about these three), however there is not particolar reason why the flavors that we see in nature should be also the eigenstates of weak interaction. So the Cabibbo idea was that weak interaction does not care about the flavors that we see in nature, but it has (for quarks) its own eigenstates. More formally:

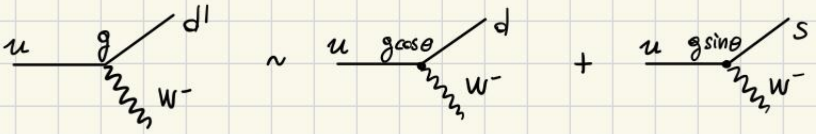
Techinically speaking the idea is that the eigenstates of weak interactions (weak Hamiltonian) are d' and s' that are linear superpositions of the flavor eigenstates d and s:

$$\boxed{\begin{aligned} d' &= \cos\theta\, d + \sin\theta\, s \\ s' &= -\sin\theta\, d + \cos\theta\, s \end{aligned}}$$

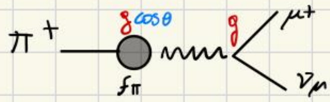


θ is known as Cabibbo angle

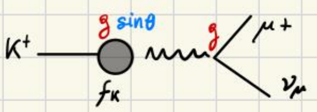
With this idea the weak interaction vertex becomes:



θ is the parameter that we have to fix from the data in order to explain the mixture that we have. Let's see how to estimate it:



$$\Rightarrow \Gamma \propto G_F^2 \cos^2\theta\, m_\mu^2 (m_\pi^2 - m_\mu^2) \frac{1}{m_\pi^3} f_\pi^2$$



$$\Rightarrow \Gamma \propto G_F^2 \sin^2\theta\, m_\mu^2 (m_K^2 - m_\mu^2) \frac{1}{m_K^3} f_K^2$$

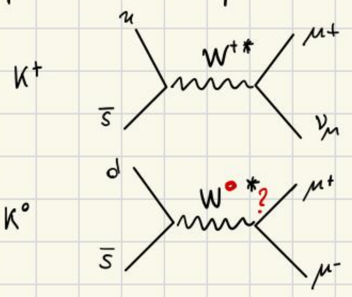
$$\frac{\Gamma(K^+ \to \mu^+ \nu_\mu)}{\Gamma(\pi^+ \to \mu^+ \nu_\mu)} = (\tan\theta)^2 \times F(m_\mu, m_\pi, m_K, f_\pi, f_K) \longrightarrow \boxed{\theta \simeq 0.26 = 13°} \qquad \begin{cases} \cos^2\theta = 0.97 \\ \sin^2\theta = 0.03 \end{cases}$$

This means that for all the various decays we always have:

$$\boxed{\Gamma(\Delta S = 1) \propto \sin^2\theta = 0.03 \quad ; \quad \Gamma(\Delta S = 0) \propto \cos^2\theta = 0.97}$$

this means that $\Gamma(\Delta S = 1)$ is always suppressed in weak interactions. This fact is known as Cabibbo suppression.

Most $\Delta S = 1$ decays explained by Cabibbo's angle. However one major problem was with leptonic decays. Let's take for example:



$\Gamma(K^+ \to \mu^+ \nu_\mu) \simeq 64\%$

Very strange: $\Gamma(K^0 \to \mu^+\mu^-) \ll \Gamma(K^+ \to \mu^+ \nu_\mu)$

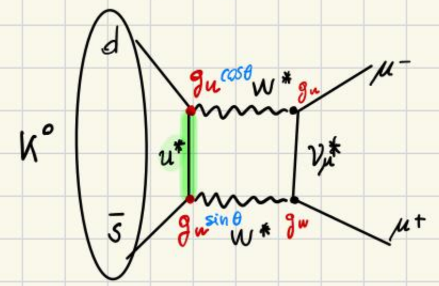
$\Gamma(K^0 \to \mu^+\mu^-) \simeq 10^{-9}$?

- Why the B.R. for the decay of the K^0 is much more suppressed?
- What is the mediator for the 2nd process? So far we've only talked about charged currents with $W^\pm$ and it is because so far we've only looked at charged vertex. In this case it is neutral and therefore

whatever it is in the middle as mediator it has no charge, so we call it, W°. Moreover it must be able to change the flavor : this tells us that trivially the mediator cannot be a γ^* (in fact $\Delta S = 1$ is not allowed in QED).

If W° does not exist we expect to have $\Gamma(K_0 \to \mu^+\mu^-) = 0$. What we find experimentally instead is that $\Gamma(K^\circ \to \mu^+\mu^-) \simeq 10^{-9}$ and therefore there are 2 possibilities : or the mass the mediator is incredibly large (not possible) or the process is actually happening without a W° through a 2nd order diagram

For the process with the K° we have the following loop diagram :



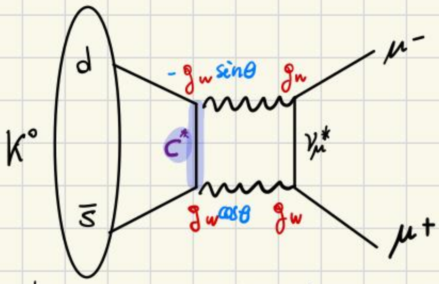
$$\mathcal{M}_u(K^\circ \to \mu^+\mu^-) \sim \underbrace{\frac{g_w^2}{m_w^2}}_{G_F^2} \underbrace{\frac{g_w^2}{m_w^2}}_{G_F^2} \sin\theta\cos\theta \sim G_F^4 \sin^2\theta \cos^2\theta$$

Doing the complete calculations of this diagram one finds that the theoretical prediction for the decay width bigger than what experimentally measured:

$$\Gamma_{th}(K^\circ \to \mu^+\mu^-) \gg \Gamma_{exp} = 10^{-9}$$

So we have a problem: if the W° does not exist then the only way in which the process can happen is via a 2nd order diagram which however lead to a bigger decay width $\to$ problem. In some sense we should have something that gives a cancellation in such a way the Γ_{th} is as small as the measured one.

The problem was solved by Glasgow - Iliopolus - Maiani (1970) with the GIM mechanism. They postulated the existence of a new quark : the Charm quark (up-like with charge $Q = +2/3$). With this new quark for the K°-decay we also have to consider the diagram:



$$\mathcal{M}_c \sim \frac{g_w^2}{M_w^2}\frac{g_w^2}{M_w^2}(-\sin\theta)(\cos\theta)\frac{1}{q_c^2}$$

$\leftarrow$ c-quark momentum

The idea at the base of GIM mechanism is that $W^\pm$ couples with $\begin{pmatrix} u \\ d' \end{pmatrix}$ and $\begin{pmatrix} c \\ s' \end{pmatrix}$.

This means that in the vertex with c, d and W we have c, s' and W. And since we have to take only the part of d prop. to s' we get a factor $(-\sin\theta)$
In the same way in the vertex with c, s and W we just have a contribution from c, s' and W, so we get a factor $(\cos\theta)$.

So with the c quark around we have a new diagram whose amplitude is exactly like the one of the previous diagram but with a minus in front and with the propagator of the charm in place of the propagator of the up. The total amplitude will be

$$\mathcal{M}_{tot} = \mathcal{M}(u\text{-quark}) + \mathcal{M}(c\text{-quark})$$

In order to have $\Gamma \sim 10^{-9}$ and therefore $\mathcal{M}_{tot} \simeq 0$ we need to have $\boxed{m_{charm} \simeq 1\text{-}3 \text{ GeV}}$

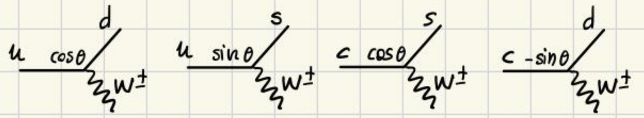
So if the charm exists (with $q_c = +\frac{2}{3}$, $m_c \simeq 1\text{-}3\,GeV$) then the decay $K^0 \to \mu^+\mu^-$ does not need a flavor changing mediator W^0 to be explained. The existence of the charm quark is something that was proven later on in 1974 with the discovery of the J/ψ

So we've understood that the interaction can happen with $\begin{pmatrix} u \\ d' \end{pmatrix}$ or with $\begin{pmatrix} c \\ s' \end{pmatrix}$. In a vertex we have in general the following structure:

$$\sim g_w V_{ij} \gamma^\mu (1-\gamma^5)$$

Where i, j are the flavors of the quarks $\{u, d, c, s\}$ and V_{ij} is a mixing matrix with the objective of taking the part only proportional to d' (if the other quark is a u) and s' (if the other quark is a c):

$$V_{ij} = \begin{array}{cc} \begin{array}{cc} d & s \end{array} & \\ \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} & \begin{array}{c} u \\ c \end{array} \end{array}$$

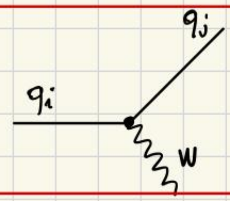


In the case of leptons instead the equivalent matrix is the identity since there is no mixing. (lepton universality).

However there was another problem: in 1964 it was observed CP violation in Kaon mesons. *In order to explain this fact* in 1973 the idea of GIM was generalized by Kobayashi and Maskawa who proposed new quarks and their interaction. They proposed the existence of a new family $\begin{pmatrix} t \\ b \end{pmatrix}$.
So they generalized the GIM mechanism saying that $W^\pm$ couples with:

$$\begin{pmatrix} u \\ d' \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix} \quad \begin{pmatrix} t \\ b' \end{pmatrix}$$

So the vertex can be generalized as:



$$\boxed{\sim g_w (V_{CKM})_{ij} \gamma^\mu (1-\gamma^5)}$$

Where we can define the so called Cabibbo - Kobayashi - Maskawa matrix (CKM matrix):

$$\boxed{V_{CKM} = \begin{array}{cc} \begin{array}{ccc} d & s & b \end{array} & \\ \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} & \begin{array}{c} u \\ c \\ t \end{array} \end{array}}$$

This matrix is a 3x3 unitary matrix parametrized by 3 real angles and by 1 complex phase.
- the complex phase could explain the CP violation
- the 3 angles can be interpreted as 3 consecutive rotations around x, y, z.

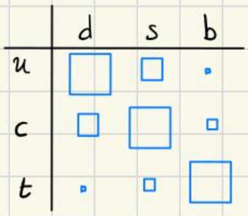
$$\begin{aligned} V_{CKM} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \end{aligned}$$

sin and cos of the Cabibbo angle

We can use the Wolfstein parametrization and write everything in terms of the Cabibbo angle with $\lambda \equiv \sin\theta_c$:

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

The further we go from the diagonal the more we have a suppression :



Why nature has this structure we have no idea

area $\propto |V_{ij}|^2$

Today we have measured the entries of the matrix. Looking at PDG we find that:

$$|V_{\mathrm{CKM}}| = \begin{pmatrix} 0.97435 \pm 0.00016 & 0.22500 \pm 0.00067 & 0.00369 \pm 0.00011 \\ 0.22486 \pm 0.00067 & 0.97349 \pm 0.00016 & 0.04182^{+0.00085}_{-0.00074} \\ 0.00857^{+0.00020}_{-0.00018} & 0.04110^{+0.00083}_{-0.00072} & 0.999118^{+0.000031}_{-0.000036} \end{pmatrix}$$

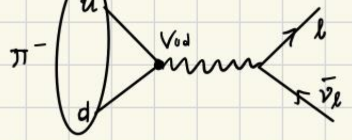
- If we look at the t,b entries they are ~ 1 and that is because the top is so heavy that it decay almost 100% of the time into on shell b and W
- Transposed entries are similar as expected by unitarity.

The CKM parameters are free parameters of the S.M. : they are not predicted by the theory but can only be measured. To experimentally measure these parameters the decay used are the following:

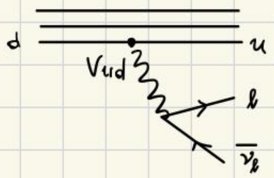
$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \pi\ell\nu \\ & K \to \pi\ell\nu & \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ B_d \leftrightarrow \overline{B}_d & B_s \leftrightarrow \overline{B}_s & \end{pmatrix}$$

- Vud measurement

In principle we could use the following two processes:



e.g. $\pi^- \to \mu^- \bar{\nu}_\mu$ $\quad \Gamma \sim |V_{ud}|^2 \times$ hadronic correction

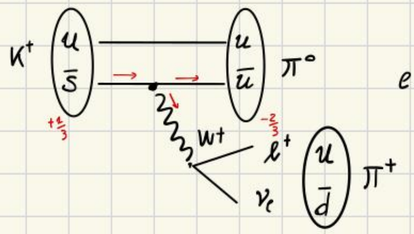


e.g $n \to p e^- \bar{\nu}_e$ $\quad \Gamma \propto |V_{ud}|^2 \times$ hadronic correction

Which of these 2 is the best in order to measure Vud? In general larger hadronic corrections in baryons, therefore mesons are preferred.

- V_{us} measurement

In principle we could use the following two processes:



e.g.

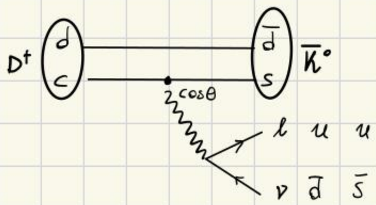
$K^+ \longrightarrow \pi^0 \mu^+ \nu_\mu \longrightarrow \Gamma \propto |V_{us}|^2 \times (\text{phase space})$

$K^+ \longrightarrow \pi^0 \pi^+ \longrightarrow \Gamma \propto |V_{us}|^2 |V_{ud}|^2 \times (\text{phase space})$

Which of these 2 is better? The phase spaces are quite similar since $m_\mu \simeq m_\pi$. So the only thing that can allows us to choose a process rather than the other is the fact that $K^+ \longrightarrow \pi^0 \mu^+ \nu_\mu$ is preferred because is not correlated to V_{ud}.

- V_{cs} measurement

In principle we could use the following 3 processes:



e.g

$D^+ \longrightarrow \bar{K}^0 e^+ \nu_e \longrightarrow \Gamma \propto |V_{cs}|^2$

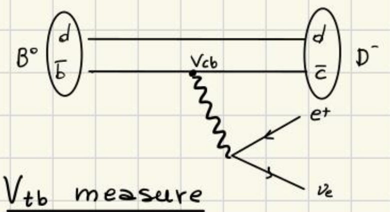
$D^+ \longrightarrow K^- \pi^+ \longrightarrow \Gamma \propto |V_{cs}|^2 |V_{ud}|^2$

$D^+ \longrightarrow K^0 K^+ \longrightarrow \Gamma \propto |V_{cs}|^2 |V_{us}|^2$

Also in this case is convenient to use the 1st one because the decay width depends only on V_{cs}.

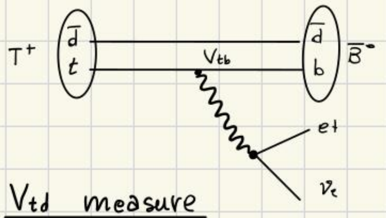
- V_{xb} and V_{tx} measurements require B mesons $B^0 = (\bar{b} d)$; $B^+ = (\bar{b} u)$

  - V_{cb} measure



$B^0 \longrightarrow D^- e^+ \nu_e \longrightarrow \Gamma \propto |V_{cb}|^2$

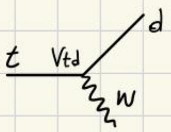
  - V_{tb} measure



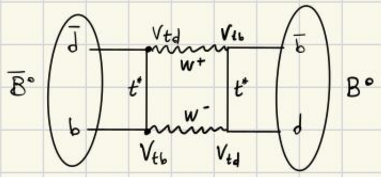
The problem is that this T^+ doesn't exist! Top indeed decays too quickly! (T^+ is toponium)

  - V_{td} measure

We need a process with the vertex



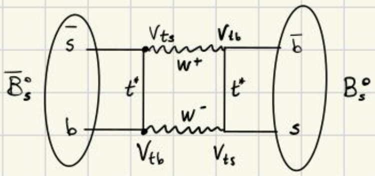
. To obtain it we need to take the virtual-top (in loop).



$\longrightarrow \Gamma \propto |V_{td}|^2 |V_{tb}|^2 \sim |V_{td}|^2$

This is a 1 body decay and in principle it shouldn't be possible. However we actually see it. How is it possible? The problem would be the momentum conservation, so the only way is to have the same mass as in this case. This effect is called $B^0\,\bar{B}^0$ oscillation. $\Delta b = 2$

- V_{ts} measure



$\longrightarrow \Gamma \propto |V_{ts}|^2 |V_{tb}|^2 \sim |V_{ts}|^2$

N.B. We'll see that flavor oscillation is the Key to understand $\cancel{CP}$.

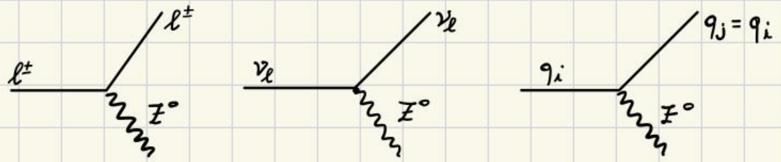
Can we have a neutral weak current?

So far we've seen that

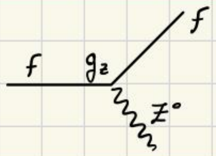
- (electrically) charged weak currents exist and can be flavor changing (in the case of hadrons)
- (electrically) neutral weak currents that are flavor changing do not exist. but until now nothing prevents to do not have flavor conserving neutral current.

Flavor conserving neutral current (Bludman 1958)

The possible vertices are:

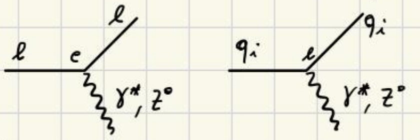


So we represent the vertex of a flavor conserving neutral current as:

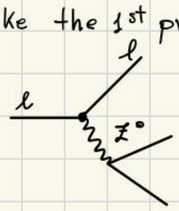


$$\boxed{f = \ell, \nu_e, q_i \qquad i = \text{flavors}}$$

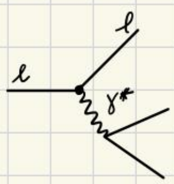
There are some difficulties with these processes mediated by a flavor conserving weak neutral current (Z^0) because the final state is the same we get with photons as mediators if we consider ℓ or q_i: (ν is not considered because $q_\nu = 0$ and QED does not work).



Let's take the 1st process:



$\sim g_Z \gamma^\mu (1-\gamma^5) \dfrac{1}{q^2 - M_Z^2} \sim \dfrac{\sqrt{\alpha_W}}{q^2 - M_Z^2}$



$\sim \dfrac{ie}{q^2} \gamma^\mu \sim \dfrac{\sqrt{\alpha_{EM}}}{q^2}$

we have that:

1) if $\alpha_{EM} > \alpha_{WEAK}$ the process is already favored
2) if $q^2 \ll M_Z^2$ the EM process dominates because $\dfrac{\sqrt{\alpha_{EM}}}{q^2} \gg \dfrac{\sqrt{\alpha_W}}{M_Z^2}$ (also if $\alpha_{EM} \sim \alpha_W$)
3) if $q^2 \simeq M_Z^2$ we go on the pole and the weak process dominates and this required high energies not reachable in 1958.

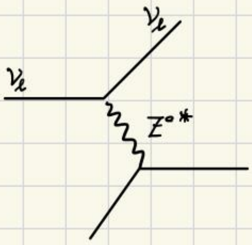
So the problem is that both EM and WEAK processes have the same initial and final state and we have to distinguish one process from the other.

In 1961 Glasgow proposed unification of Weak and EM interaction (mixture of γ and Z^0 fields).
In 1967 Weimberg – Salam proposed Glasgow's theory as a spontaneusly broken Gauge theory : $SU(2)_{WEAK} \times U(1)_{EM}$
In 1971 t'Hooft proved that this theory is renormalizible
If the theory had been correct they would have had to find experimental evidence for W and Z^0 as happened
In 1979 Glasgow – Weinberg – Salam received the Nobel.

Indirect proof of Z^0

If we use e^-, μ^- or quarks we're going to lose against the EM process as we saw. Therefore the solution is to use neutrinos (or antineutrinos):

$\nu_\ell + X \longrightarrow \nu_\ell + X$ where $X = e^-, p, n$



The experimental challenges are:

1) Have a beam of neutrinos
2) Have an high intensity
3) Have a focused beam
4) Very small cross section $\sigma \sim 10^{-55} cm^2$

N.B. Reines and Cowan used neutrinos by putting the experiment near a reactor. However they had ~MeV neutrinos and a $q^2 \sim$ MeV is to small for our goals, moreover we need to have the momentum under control. (while instead they had neutrinos going in all directions).
So, we need to have a beam of neutrinos: it is not something easy to obtain, it took about 15 years to produce it and the discovery came in 1973 at Cern.

Experiment @ CERN 1973

To produce the ν-beam the idea was a proton beam on a target of Beryllium and the things that can happen are:

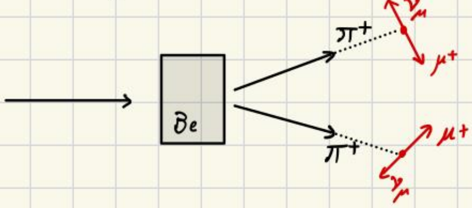
$$p + Be \longrightarrow \pi^+ + X \;;\; \pi^- + X \;;\; K^+ K^- + X \;;\; \dots \quad \Rightarrow \quad p + Be \longrightarrow \{\pi^\pm, K^\pm\} + X$$

(There can also be baryons in the final state if we put enough energy in the initial state).
We want to obtain π in the final state because they have an ultra-preferred decay channel ~99%:

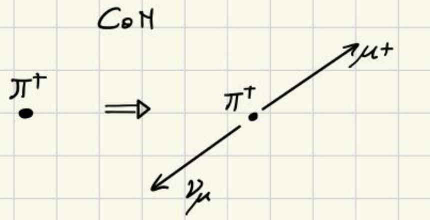
$\pi^\pm \longrightarrow \mu^\pm \overline{\nu}_\mu$ ~ 99% $\longrightarrow$ we can produce a beam of ν pore at ~99% with high intensity.

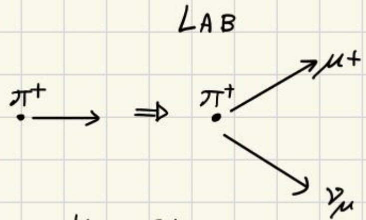
$K^\pm \longrightarrow \mu^\pm \overline{\nu}_\mu$ ~ 60%
$K^+ \longrightarrow \pi^0 \mu^+ \nu_\mu$ ~ 3%
$K^+ \longrightarrow \pi^0 e^+ \nu_e$ ~ 5%

Moreover that decay is such that $\tau_\pi \sim 10^{-8} s \longrightarrow \ell_\pi \sim \beta\gamma c \tau_\pi \sim O(m)$: this means that pions flight meters before decaying.
However we have a problem given by the fact that we can't control the kinematics of the pion

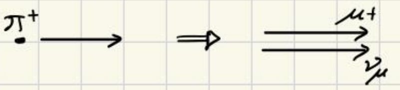


$\longrightarrow$ particles are spread out. In the CoM of the π by virtue of the fact that spin of π is zero, the decay is isotropic



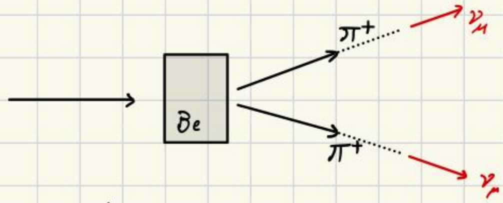


If $\beta_\pi^{LAB} > \beta_\nu^*$ in the lab frame we have the following situation:



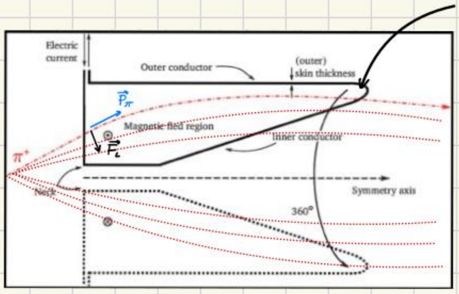
ν_μ goes forward in the same direction of π in the lab frame.

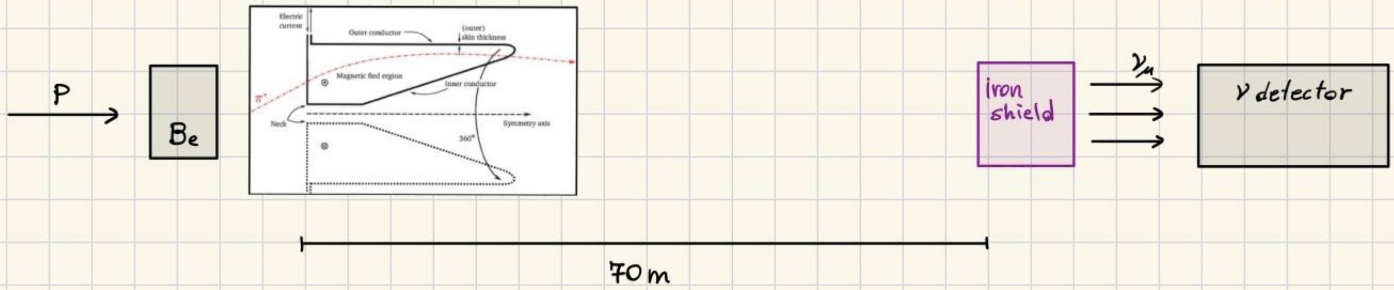
So the idea is have a very high energetic proton beam and hence high energetic pions in order to have $\beta_\pi^{LAB} > \beta_\nu^*$ and therefore have ν_μ going in the same direction as the π in the lab frame.



Now we have to focuse the ν_μ and to do so we have to focus the pions. The problem was solved by Van der Meer. He invented this horn:

switching the sign of $\vec{B}$ we select π^- and hence $\bar{\nu}$





The iron shield is there to stop

- π^+, K^+, baryons, n, p, ... via nuclear interaction, ionization
- $e^\pm$ via ionization, EM shower
- $\mu^\pm$ via ionization

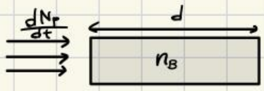
Knowing the interaction lenght λ_{int}, the number of particles surviving the shield goes like $N_{surv} \sim e^{-x/\lambda_{int}}$.

So in the end we're able to abtain a beam of neutrinos, we don't know their momentum but we know with a certain accuracy its direction of flight

The detector used is Gargamelle:

- Steel cylinder filled with freon liquid CF_3Br (liquid)
- It is a bubble chamber
- Presence of a 2T magnetic field
- We took picture of the events (expected low rate)

Observation : we know that $\frac{dN_r}{dt} = \frac{dN_p}{dt} \sigma n_B d$

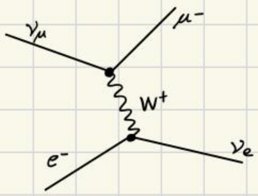


- $\sigma_\nu \ll 1$ fixed
- to increase $\frac{dN_p}{dt}$ we need an high intensity beam → smash as many protons per unit of time as we can.
- d is as long as we can
- we like to have n_B large
- The choice of freon was motivated by the fact that it was really hot (just below the boiling point) and it was easy to take pictures with it.

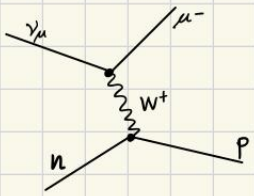
Let's see what kind of events we expect. The ν can interact with ordinary matter, so we expect events of the kind:

$$\nu + \begin{Bmatrix} e \\ p \\ n \end{Bmatrix} \longrightarrow \nu + X$$

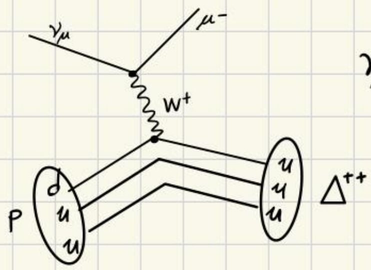
Charged current process (CC)



$\nu_\mu + e^- \longrightarrow \mu^- + \nu_e$



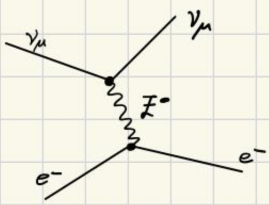
$\nu_\mu + n \longrightarrow \mu^- + p/X$



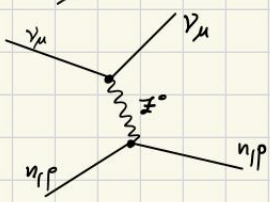
$\nu_\mu + p \longrightarrow \mu^- + \Delta^{++}/X$

(CC) events have μ^- in the final state.
Such events are background but we can exploit the μ^- in order to cut off them.

If the Z^0 exists → neutral current processes (NC) (observable)

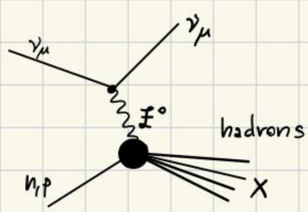


$\nu_\mu + e^- \longrightarrow \nu_\mu + e^-$



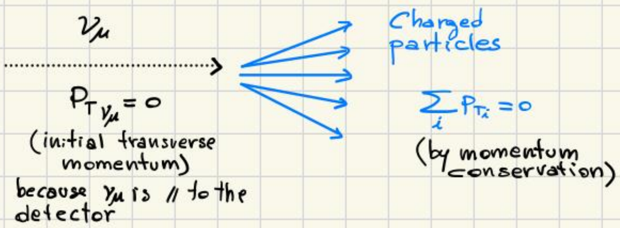
$\nu_\mu + n/p \longrightarrow \nu_\mu + n/p$

ELASTIC SCATTERINGS



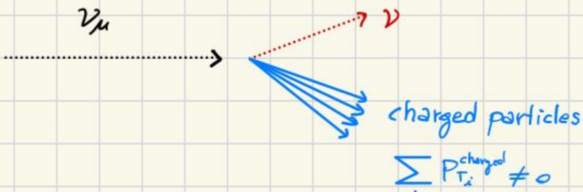
$\nu_\mu + n/p \longrightarrow \nu_\mu + \text{hadrons}$ INELASTIC SCATTERING

Those are the processes we'd like to use to distinguish the (NC) from the (CC). Then there is an additional difference between the CC and the NC that we can understand by looking at momentum conservation. With a CC in the final state we always have all charged particles (except for the case $\nu_\mu + e^- \longrightarrow \mu^- + \nu_e$ that we can still distinguish because of the μ).



$\longrightarrow$ this means that for CC processes we could also check this constraint

Instead for a NC process there is also a ν in the final state (and no muons)

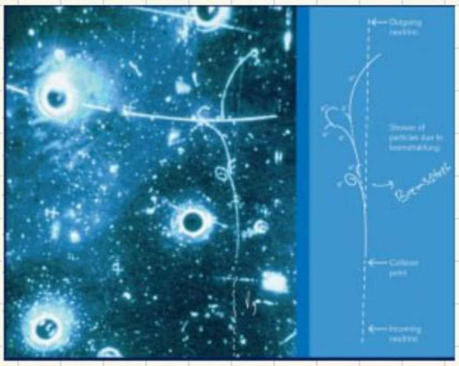


$\longrightarrow$ this means that a NC process has un $\sum P_{T_i}^{charged} \neq 0$ that we could use to distinguish from a CC process

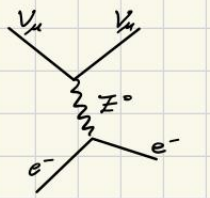
In conclusion, to distinguish CC from NC we basically see 2 things:

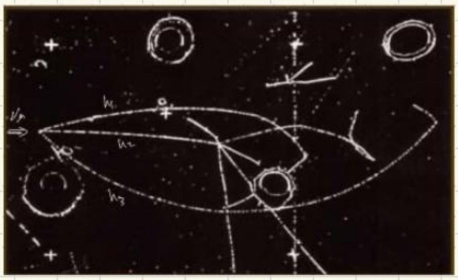
1) presence of μ
2) unbalance of $\sum_i P_{T_i}^{charged}$

<u>What did they see?</u>

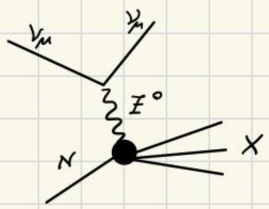


$\nu_\mu + e^- \longrightarrow \nu_\mu + e^-$





$\nu_\mu + N \longrightarrow \nu_\mu + 3\ \text{hadrons}$



The events that they found were:

- with a ν_μ beam: 102 NC, 428 CC, 15 n background
- with a $\bar{\nu}_\mu$ beam: 64 NC, 148 CC, 12 n background

$\longrightarrow \dfrac{NC}{CC} \simeq \dfrac{1}{3} \neq 0$

So we can say that:

1) $\dfrac{NC}{CC} \neq 0 \longrightarrow$ neutral current exists

2) $\dfrac{NC}{CC} \simeq 1/3 \longrightarrow g_W \neq g_Z$

The dominant CC diagram has an amplitude of order $\sim G_F$ $\longrightarrow$ we should se a suppression factor of the order of $\sim G_F^2$ but since we have $\frac{1}{3} \gg G_F^2$ we then conclude that the neutral current exists.
N.B. The fact that $CC \simeq 3\ NC$ cannot be explained with higher order processes. it is too much.

Electro weak theory

Glasgow - Weinberg - Salam formulated the so called Electro-weak theory $SU(2)_W \times U(1)_{EM}$ with the boson W^1, W^2, W^3 coming from $SU(2)_W$ and the boson B coming from $U(1)_{EM}$.

- W^1 and W^2 provide us 2 charged heavy bosons

$$\boxed{W^{\pm} = W^1 \pm i W^2}$$

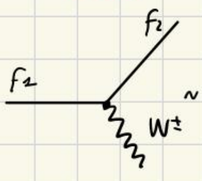
- W^3 and B provide us the 2 physical bosons Z^0 and $A(\gamma)$, by a mixture represented by:

$$\boxed{\begin{pmatrix} Z^0 \\ A \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W^3 \\ B \end{pmatrix}}$$

θ_W is the Weinberg mixing angle (not fixed by theory, must be measured)

The structure of the Weak interaction and the obvious E.M. interaction are:

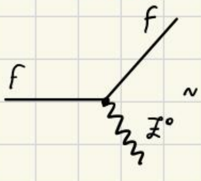
C.C.



$\sim g_W \gamma^\mu (1-\gamma^5)$

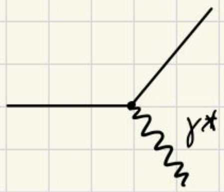
$f_{1,2} = \ell^{\pm}, \nu_\ell, \bar{\nu}_\ell, q_i$
i = flavors of quarks

N.C.



$\sim g_Z \gamma^\mu (C_V^f - C_A^f \gamma^5)$

f = fermion



$g_e \gamma^\mu$

C_V^f and C_A^f are respectively the vector and the axial coupling and they depend on the lepton / quark flavor. Glasgow - Weinberg - Salam predicted these values (Nobel 1970). The couplings predicted by the theory were:

f	c_V	c_A
ν_e, ν_μ, ν_τ	$\frac{1}{2}$	$\frac{1}{2}$
e^-, μ^-, τ^-	$-\frac{1}{2} + 2\sin^2\theta_w$	$-\frac{1}{2}$
u, c, t	$\frac{1}{2} - \frac{4}{3}\sin^2\theta_w$	$\frac{1}{2}$
d, s, b	$-\frac{1}{2} + \frac{2}{3}\sin^2\theta_w$	$-\frac{1}{2}$

What the theory predicts is also that:

$$\boxed{g_W = \frac{g_e}{\sin\theta_W}, \quad g_Z = \frac{g_e}{\sin\theta_W \cos\theta_W} \quad g_e = \frac{e}{\sqrt{4\pi}}}$$

So there is not a fundamental difference between the weak and the EM coupling but a difference given by the Weinberg mixing angle.
Another prediction of the theory is that: $\boxed{M_W = M_Z \cos\theta_W}$

so the experimental exstimation of M_W / M_Z gives an exstimation of the Weinberg angle

$$\boxed{\theta_W \simeq 29^\circ, \ \sin^2\theta_W \simeq 0.23.}$$

Later t'Hooft proved that it's renormalizable and he also got the Nobel prize.

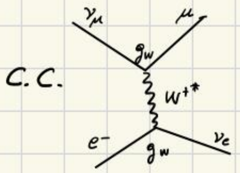
To verify GWS theory we need to measure:

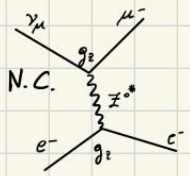
1) quantities sensitive to C_V, C_A e.g. g, $\cos\theta_W$, $\sin\theta_W$.
2) M_W, M_Z

1 MEASURE C_V, C_A

We can consider the 2 processes:

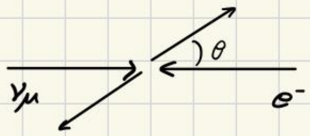
$\nu_\mu + e^- \longrightarrow \mu^- + \nu_e$ $\qquad\qquad$ $\nu_\mu + e^- \longrightarrow e^- + \nu_\mu$





$\longrightarrow \sigma_{CC} \sim \sigma(g_W) = \sigma(g, \sin\theta_W)$ $\qquad$ $\longrightarrow \sigma_{NC} \sim \sigma(g_Z) = \sigma(g, \sin\theta_W, \cos\theta_W)$

To calculate the cross section then we put ourselves in the CoM frame:



2 physical parameters: θ, E_{beam}

if $q^2 \ll M_Z^2, M_W^2$ doing the complete computation one can find that:

$$\sigma(NC) \sim \left(\frac{g_Z}{M_Z}\right)^2 E^2 (C_V^2 + C_A^2 + C_V C_A)$$

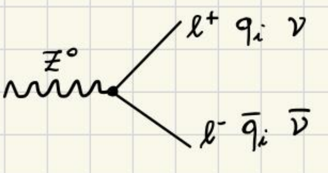
then one can find that:

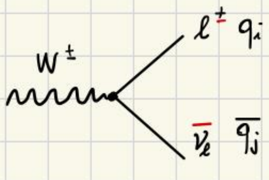
$$\frac{\sigma_{NC}}{\sigma_{CC}} = \frac{\sigma(\nu_\mu + e^- \longrightarrow \nu_\mu + e^-)}{\sigma(\nu_\mu + e^- \longrightarrow \mu^- + \nu_e)} = \frac{1}{4} - \sin^2\theta_W + \frac{4}{3}\sin^2\theta_W \simeq 0.09 \longrightarrow \boxed{\frac{\sigma_{NC}}{\sigma_{CC}} \simeq 0.09}$$

Instead experimentally the result is $\boxed{\frac{\sigma_{NC}}{\sigma_{CC}} \simeq 0.11}$: the indirect proof of EW theory seems to work!

2 PRODUCE W AND Z ON SHELL (DIRECT EVIDENCE)

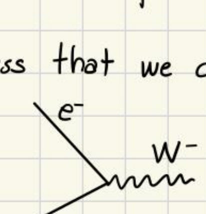
The processes that we know that can happen are:



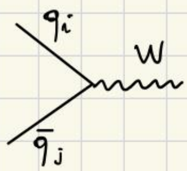


From Fermi theory $G_F \simeq 10^{-5}\,\text{GeV}^{-2}$ and it was estimated that $M_W, M_Z \simeq 80-100$ GeV.
If a decay can happen then also the opposite process can happen, as long as there is enough energy: this tells us how we can produce W and Z.

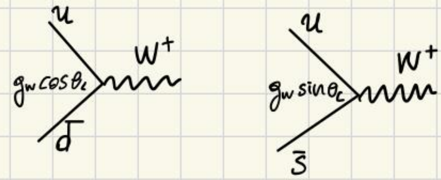
To produce a W on shell a process that we could use with leptons is the following:



however this process is hopeless: $\bar{\nu}_e$ can be generated by $n \longrightarrow p e^- \bar{\nu}_e$ but we're not able to accelerate neither the neutron neither the antineutrino. Moreover it require very focused beams and an e^- beam's energy too high (because $\bar{\nu}_e$ from n decay has ~ 3 MeV). So the only accessible way is:



e.g.

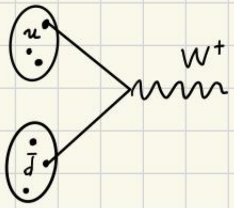


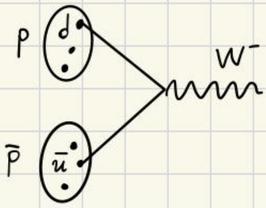
We could take in practice $\pi^+ : (u\bar{d})$ or $\pi^- : (\bar{u}d)$, which could be produced through:

$$p + p \longrightarrow \pi^+\pi^- p p$$

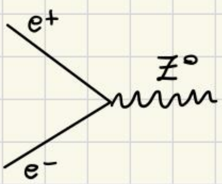
However $\pi^\pm$ are unstable with $\tau \sim 10^{-8}$s and this means that there is not enough time to collect them. Therefore we could use baryons like $p : (uud)$ or $n : (ddu)$; in particular p is better because I can accelerate it. Of course since I need also an antiquark to obtain it we use also antiprotons $\bar{p} : (\bar{u}\bar{u}\bar{d})$. So what we do is to collide protons and antiprotons:

$$\boxed{p + \bar{p} \longrightarrow W^+ + X \qquad p + \bar{p} \longrightarrow W^- + X}$$





To produce Z^0 on shell instead one strategy would be to use 2 colliding beams of e^+ and e^-:

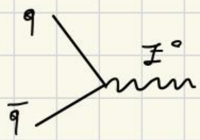


$$\longrightarrow \quad E_{beam} = \frac{M_Z}{2} \simeq 45 \text{ GeV} \quad \text{(challenging)}$$

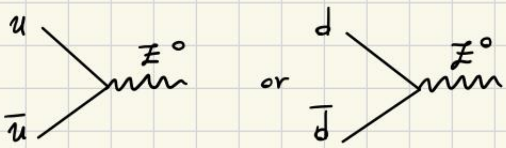
However with electrons we have the problem of energy loss via Bremstrahlung with a prob. of emission of a γ that goes like $\sim \frac{1}{m^5}$ (practically it happens only with electrons). An idea could be make a larger ring in order to minimize this radiative energy loss and this would be expensive. Another possibility could be with muons: the advantage is that they won't have this radiation problem and the radius of the ring could be smaller. However at that time having two colliding beams of μ^+ and μ^- was challenging because of muon's decay.

The last lepton possibility was a $\tau^+\tau^-$ collider but it cannot be done because the τ lifetime is too short.

Therefore also in this case the most practical way to produce it is through quarks:

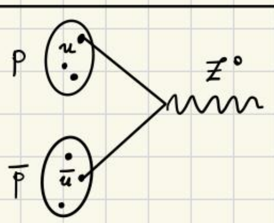


e.g.



Also in this case the best thing to do is to collide protons and anti-protons

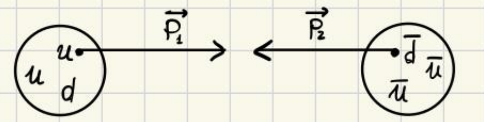
$$\boxed{p + \bar{p} \longrightarrow Z^0 + X}$$



The experimental challenge was to produce many $\bar{p}$ and then to accelerate them. To produce antiprotons we can collide protons beams : $p + p \longrightarrow p+p+\bar{p}+p$.
The problem is that we would like to have this $\bar{p}$ focused : the problem was solved by Simon Van der Meer who invented the Stochastic Cooling of antiprotons. The word cooling refers to the minimization of the transverse momentum of the anti-proton is such a way that $\vec{P}_{\bar{p}} \simeq \vec{P}_L$.
This technique is still used today.

Back in the 1970's they had SPS (Super Proton Synchroton) at CERN. Then in 1976 Rubbia had the idea of turning SPS into S$p\bar{p}$S (Super Proton antiProton Synchrotron).
Once antiprotons are produced we can accelerate them. Of course we need to change the sign of the magnetic field in order to accelerate them in opposite direction and then make them collide.
In UA1 (Underground Area 1) and UA2 there were 2 detectors and each one of them was able to see $p\bar{p}$ collisions.
For this experiment in 1984 Rubbia and Van der Meer got the Nobel prize.

Since we are colliding composite particles we don't know exactly the center of mass energy $\sqrt{s}$



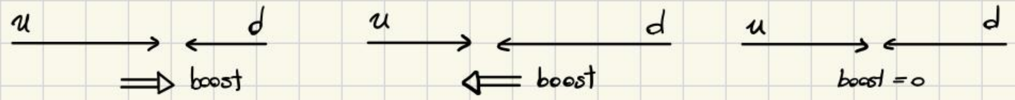
$P_1 = (E_1, E_1, 0, 0)$ $\qquad P_2 = (E_2, -E_2, 0, 0) \longrightarrow \sqrt{s} = \sqrt{(P_1+P_2)^2} = \sqrt{(E_1+E_2, E_1-E_2, 0, 0)^2} \simeq E_1+E_2$

The energies of the quarks E_1 and E_2 are $\neq E_{beam}$. These 2 energies are unknown but roughly we can say that:

$$E_1, E_2 = x\, E_{beam} \simeq \frac{1}{3} E_{beam} \qquad \text{where } x \in [0,1] \text{ is the parton density function}$$

$\longrightarrow \sqrt{S_{u\bar{d}}} \simeq \frac{1}{3}\sqrt{S_{p\bar{p}}}$ where $\sqrt{S_{p\bar{p}}} = E_p + E_{\bar{p}} = 540$ GeV $\longrightarrow$ $\boxed{\sqrt{S_{q_i\bar{q}_j}} \simeq 170 \text{ GeV} > m_W, m_Z}$ (enough energy to produce them)

The another thing to take into account is the fact that we don't know the Lorentz boost in each collision because we do not know the momentum of each parton:



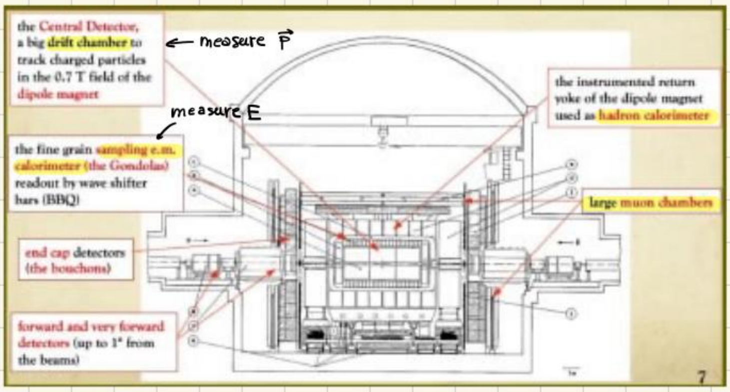
However we know that the boost cannot be in the transverse direction $(P_p + P_{\bar{p}})_T \simeq 0$ (This is not completely true because there's the Fermi energy but negligible respect to 540 GeV)
That is because in the initial state the transverse momentum is exactly 0 $\vec{P}_T = 0$. and from momentum conservation $\sum_i \vec{P}_{i_T} = 0$

So, the processes that we'll look for are:

$$\boxed{\begin{array}{l} p+\bar{p} \longrightarrow W \text{ or } Z^0 + X \\ \qquad\qquad Z^0 \longrightarrow \ell^+\ell^-,\ q_i\bar{q}_i \\ \qquad\qquad W \longrightarrow \ell\bar{\nu},\ q_i\bar{q}_j \end{array}}$$

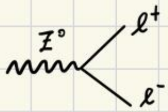
X: hadronization of the broken p and $\bar{p}$ (can be $\pi^\pm, \pi^0, K^\pm, K^0, p, \bar{p}$...) (by the spectator quarks)

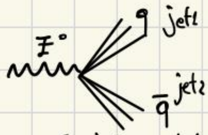
Let's see the picture of the detector used UA1. The goal was to measure the transverse momentum of all particles (to say if ther was unbalancement in the event) and to measure the only observable that does not depend on the unknowed boost : the invariant mass.



How to find Z^0

In principle we could use $Z^0 \to \ell^+\ell^-$ or $Z^0 \to q\bar{q}$.

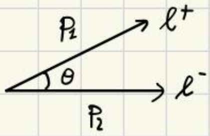




However the lepton process is preferred due to the presence, in the 2nd process, of jets which imply a worse experimental resolution of $\sqrt{s}$. an require their reconstruction. N.B. The jets there exist because of the hadronization.

We produce $Z^0 \to \ell^+\ell^-$. In the rest frame we have $\xleftarrow{\ell^+} Z^0 \xrightarrow{\ell^-}$ $E_{max} = \frac{M_Z}{2} \gg m_\mu, m_e$.

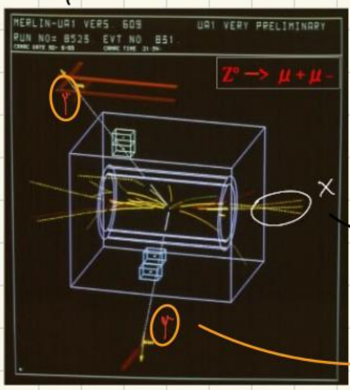
So we can treat μ and e^- as massless particles. In the LAB frame the boost is unknown and what we can do is to measure the magnitude $|\vec{P}|$ and the angle of emission θ of $\ell^+\ell^-$.



Measuring P_1 an P_2 in the lab we can estimate the invariant mass as:

$$\sqrt{s_{\ell^+\ell^-}} = \sqrt{(P_1+P_2)^2} = \sqrt{P_1^2 + P_2^2 + 2\, P_1 \cdot P_2} = \sqrt{\underset{\simeq 0}{m_1^2} + \underset{\simeq 0}{m_2^2} + 2\left(E_1E_2 - F_1F_2\cos\theta\right)} \simeq \sqrt{2E_1E_2(1-\cos\theta)}$$

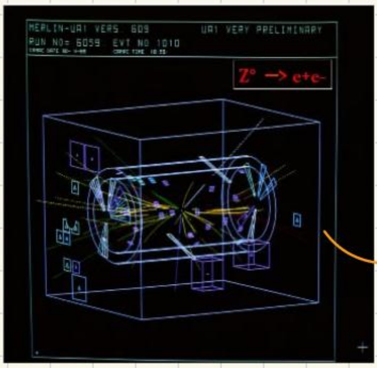
Let's report some events where they saw the Z:



$$p + \bar{p} \longrightarrow Z^0 + X$$
$$\hookrightarrow \mu^+\mu^-$$

hadronization products

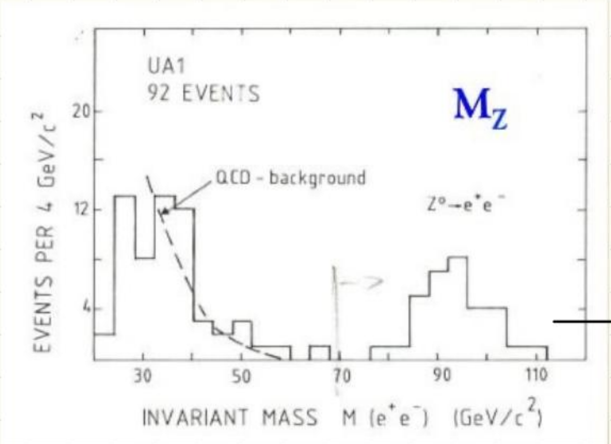
looking at the muons we can say that the Z was going to the left



$$p + \bar{p} \longrightarrow Z^0 + X$$
$$\hookrightarrow e^+e^-$$

in this case with electrons we don't have tracks far away (they are due only to muons)

Putting all together we obtain the following invariant-mass plot:

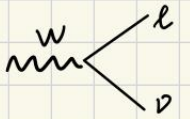


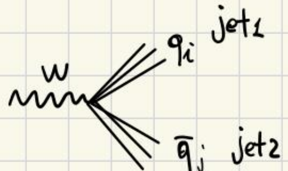
- they measured the energy in the calorimeter
- they measured the opening angle
- so they reconstructed the 4 momentum and from it they found the invariant mass. they put in the plot.

$\longrightarrow$ high evidence for Z^0 far from b.g. region

How to find W

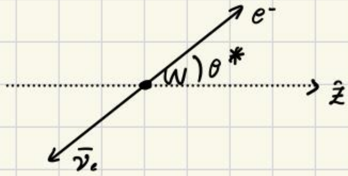
Also in this case we could use $W \longrightarrow \ell \nu$ or $W \longrightarrow q_i \bar{q}_j$





And also in this case we prefer the lepton process due to the worse resolution on $\sqrt{s}$ in the 2nd process, due to the presence of jets.

Let's focus, therefore in the lepton process. There is 1 track (e^- or μ^-) that we can clearly see and easily measure its momentum and energy. Then there is some missing energy due to ν that we can't measure. In order to reconstruct the invariant mass we need to extract some informations about ν.
In the rest frame we have:

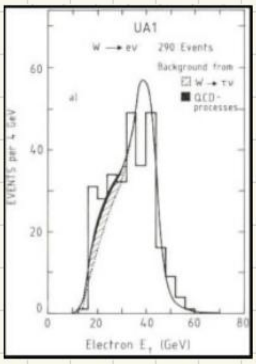


- $E_\ell^* = E_{\nu}^* = \frac{M_W}{2} \longrightarrow P_\ell^* \simeq \frac{M_W}{2} \longrightarrow \boxed{P_{\ell x}^* = \frac{M_W}{2} \sin\theta^* = P_{\ell x}^{LAB}}$
(since P_x is $\perp$ boost)

- $$\frac{dn}{dP_{\ell x}} = \frac{dn}{d\theta^*}\frac{d\theta^*}{dP_{\ell x}} = \frac{dn}{d\theta^*}\frac{1}{\frac{dP_{\ell x}}{d\theta^*}} = \frac{dn}{d\theta^*}\frac{1}{\frac{M_W}{2}\cos\theta^*} = \frac{dn}{d\theta^*}\frac{1}{\frac{M_W}{2}\sqrt{1-\sin^2\theta^*}} = \frac{dn}{d\theta^*}\frac{1}{\frac{M_W}{2}\sqrt{1-\frac{P_{\ell x}}{M_W/2}}}$$

$$\longrightarrow \boxed{\frac{dn}{dP_{\ell x}} = \frac{dn}{d\theta^*}\frac{1}{\sqrt{\left(\frac{M_W}{2}\right)^2-(P_{\ell x})^2}}}$$

This is an histogram that we can experimentally measure, we just have to see how many events we find for a given value of $P_{\ell x}$ of the lepton.
We have a pole at $\frac{M_W}{2}$ so we'll expect to have a maximum at the values around $\frac{M_W}{2}$. What they experimentally saw is:

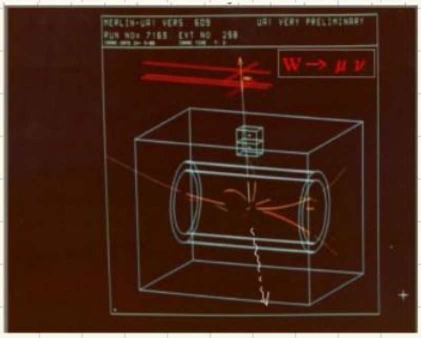


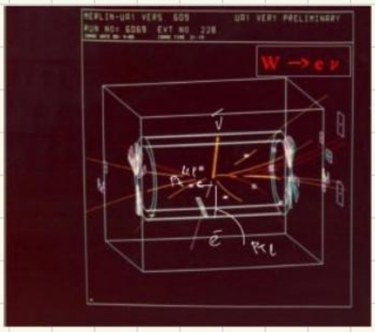
The main source of background comes from the process:

$$W^- \longrightarrow \tau^- \bar{\nu}_\tau$$
$$\tau^- \longrightarrow e^- \bar{\nu}_e \nu_\tau$$

this e^- is the one that we want to cut off. To do that we use the fact that it has less energy because there are 2 ν in this case.

Let's report some events where they saw a W:



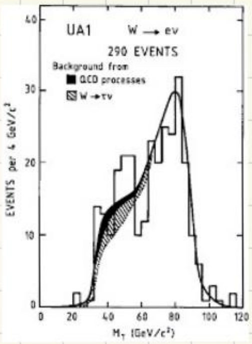


We're able to draw an arrow for the "invisible" neutrino (like we did in the figures above) because with $\vec{P}_T = (P_x, P_y)$ we have:

$$\sum_i^{\text{final particles}} \vec{P}_{Ti} = \vec{0} \longrightarrow \sum_i^{\text{charged p.}} \vec{P}_{Ti} + \vec{P}_{T\nu} = \vec{0} \longrightarrow \boxed{\vec{P}_{T\nu} = -\sum_i^{\text{charged p.}} \vec{P}_{Ti}} \quad \text{(best estimate for } \vec{P}_{T\nu}\text{)}$$

Using it we're able to compute the invariant mass: $\boxed{\sqrt{s_T} = \sqrt{(P_e + P_{T\nu})^2}}$

What they found is that:



- if the neutrino is in the transverse plane then we can reconstruct the full invariant mass, otherwise just by looking at the invariant mass in the transverse plane we're missing a part of the energy. That is why we see a "shoulder" on the left.

So, to conclude, in this way they were able to prove that W an Z existed and in 1984 they got the Nobel prize.

CP violation

Experimental methods

B meson CP violation:

- B factory and direct proof
- B0-B0bar mixing
- Interference between mixing and decays

CP VIOLATION

To observe CP violation we can use the following methods:

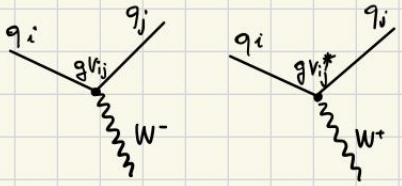
INDIRECT CP VIOLATION: this method exploit the mesons oscillations processes like $B^0 \leftrightarrow \bar{B}^0$ or $K^0 \leftrightarrow \bar{K}^0$ seeing that such transformations do not occur with exactly the same probability in both directions. (same method used for the discovery of CP violation in K^0; 1964)

DIRECT CP VIOLATION: this method is based on the searching for processes where CP eigenstate $\longrightarrow$ CP eigenstate or seeing that for the decay processes $A \to f$ and $\bar{A} \to \bar{f}$ the probability are not the same, suggesting that matter and antimatter behave differently.

$\text{CP eigenstate}_{(-1)} \longrightarrow \text{CP eigenstate}_{(+1)}$

In 1973 was proposed, as we know, the KM mechanism for the CP violation in SM. This KM mechanism works with the CKM matrix: a 3x3 unitary matrix parametrized by 3 euler angles and 1 complex phase (describing the quark mixing)

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{td} \end{pmatrix}$$



As we said the V_{CKM} is a unitary matrix, therefore

$$\boxed{V_{CKM}^{\dagger} V_{CKM} = \mathbb{1}}\ .$$

This relation gives us 9 conditions on the elements of the CKM matrix. The 3 easy conditions are the ones we get from the diagonal (related to the relation $\sin^2\theta_c + \cos^2\theta_c = 1$):

$$\boxed{\begin{array}{ccccccc} |V_{ud}|^2 & + & |V_{us}|^2 & + & |V_{ub}|^2 & = & 1 \\ |V_{cd}|^2 & + & |V_{cs}|^2 & + & |V_{cb}|^2 & = & 1 \\ |V_{td}|^2 & + & |V_{ts}|^2 & + & |V_{tb}|^2 & = & 1 \end{array}}$$

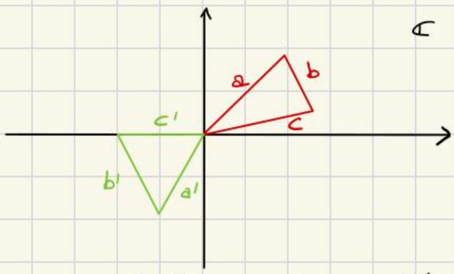
From the off diagonal terms the unitary relation gives us:

$V_{ud}^* V_{us} + V_{cd}^* V_{cs} + V_{td}^* V_{ts} = 0$	$\lambda\ \lambda\ \lambda^5$
$\boxed{V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0}$	$\lambda^3\ \lambda^3\ \lambda^3$
$V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb} = 0$	$\lambda^4\ \lambda^2\ \lambda^2$
$V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = 0$	$\lambda^3\ \lambda^3\ \lambda^3$
$V_{td}^* V_{cd} + V_{ts}^* V_{cs} + V_{tb}^* V_{cb} = 0$	$\lambda^4\ \lambda^2\ \lambda^2$
$V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb} = 0$	$\lambda\ \lambda\ \lambda^5$

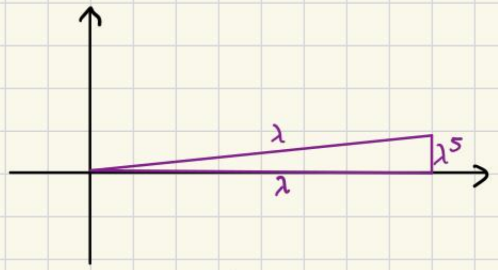
each addenum as a magnitude of a specific power of λ.

$\lambda \equiv \sin\theta_c \simeq 0.22$

These are relations between complex numbers. In each row we have the sum of 3 complex numbers (3 vectors in the complex plane) which gives zero. So these are triangular relation:



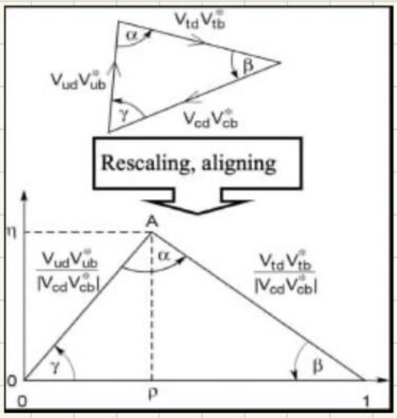
As we can see from the magnitude of the addenda (expressed in powers of λ) there are relations which are very unbalanced. For example in the first row:



So it is not easy to measure all the sides with the same experimental resolution. Therefore the interesting processes are those in which all the sides are of the same order, for example like the 2^{nd} relation where we have all sides are of order λ^3 ; moreover it contains also terms of the kind V_{ib} , so we can test it using B mesons.

KM predicted the existence of a complex phase in the CKM matrix in order to have a CP violation. We need, therefore, an experimental proof of the existence of at least one complex phase and from what we've seen above the best thing to do is to search for it using B mesons.
So we look at the interesting relation that we already noted above:

$$\boxed{V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0}$$



In the triangle we can see that we have to measure the 3 angles and the η and ρ of the Wolfstein parametrization.

These angles corresponds to the following phases of the CKM matrix

$$\begin{pmatrix} 1 & 1 & e^{-i\gamma} \\ 1 & 1 & 1 \\ e^{-i\beta} & 1 & 1 \end{pmatrix}$$

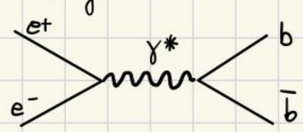
If we measure $\beta, \gamma \neq 0$, then CKM has a complex phase and therefore we have CP violation

B FACTORY: How to produce $B^0/\bar{B}^0$?

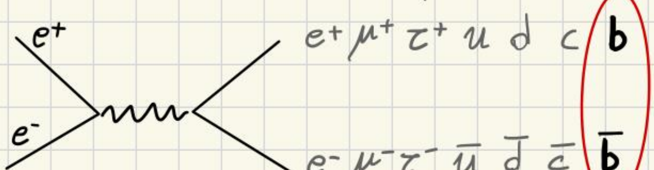
The idea is to use, as we said $B^0/\bar{B}^0$ mesons and we need a lot of them. We could use the following method:

- B^0 is made of $(\bar{b}d)$ and $\bar{B}^0$ of $(b\bar{d})$
- First of all we produce the bound state Υ $(b\bar{b})$ through e^+e^- collisions:

$$e^+e^- \longrightarrow b\bar{b} \text{ (bound state)}$$

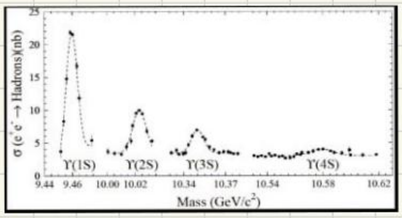


Of course $b\bar{b}$ is not the only channel:



$$\frac{\sigma(e^+e^- \longrightarrow b\bar{b})}{\sigma(e^+e^- \longrightarrow tot)} \simeq \frac{1.2 nb}{3.5 nb} \simeq \frac{1}{4} \quad \text{not bad!}$$

Why Υ ? Because it has the possibility to decay into B mesons! First of all let's see its spectrum:

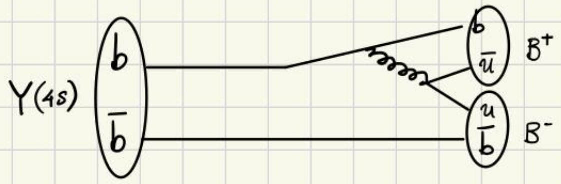


For our purposes we want to use the $\Upsilon(4S)$ resonance since it has enough mass in order to have a $Q_{value} > 0$ in the decay process into $B^0\bar{B}^0$ (or B^+B^-).

$$\boxed{Q\,(\Upsilon \to B^0\bar{B}^0 / B^+B^-) = m_{\Upsilon(4S)} - 2m_B > 0}$$

In order to do that we have to set the c.o.m energy to $\boxed{\sqrt{s} = 10.58 \text{ GeV}}$.
The way in which $\Upsilon(4S)$ can do this process is:

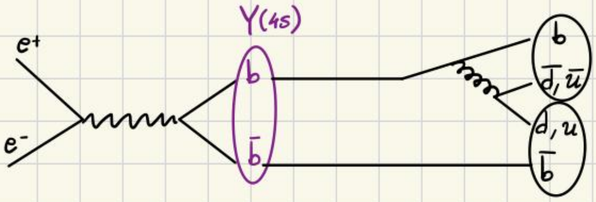
or



This is called B-factory at $\Upsilon(4S)$. To recap, we have the following process:

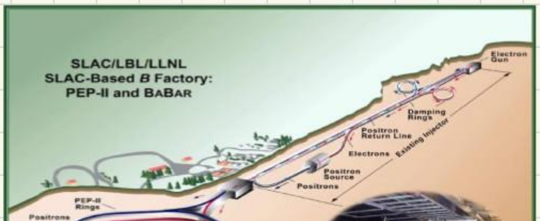
$$e^+e^- \longrightarrow b\bar{b} \equiv \Upsilon(4S) \longrightarrow \begin{cases} B^0\bar{B}^0 & \sim 50\% \\ B^+B^- & \sim 50\% \end{cases}$$

this means that $\dfrac{\sigma(e^+e^- \to b\bar{b} \to B^0\bar{B}^0)}{\sigma(e^+e^- \to tot)} \simeq \dfrac{1}{8}$



$$Q = m_{\Upsilon(4S)} - 2m_B = 10.58 - 2 \times 5.28 \simeq 0.02 > 0$$

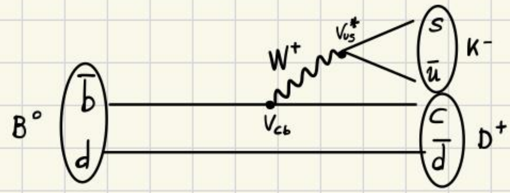
At Slac they made this B factory and it was working for ≈ 10 years. It was a linear collider, approximately 2 km long with underground detectors.



There was only one experiment running, Babar, in which they looked for $B^0\bar{B}^0$ decays. (looking for CP violations)

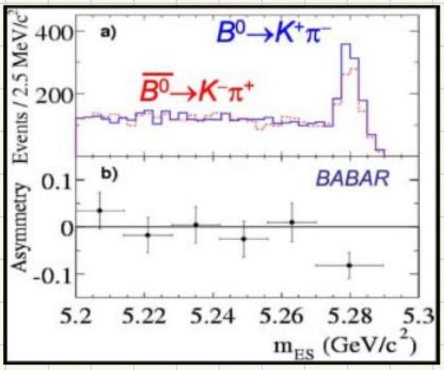
One way to look for CP violations in a direct way is to measure the number of events of the 2 following CP conjugate decays:

$$\boxed{B^0 \longrightarrow D^- K^+} \quad \xleftrightarrow{CP} \quad \boxed{\bar{B}^0 \longrightarrow D^+ K^-}$$



Actually for "Babar" <u>they looked for π instead of D</u> and the reason is that since $N = \sigma \cdot \mathcal{L} \cdot \Delta T$ where $\sigma \propto |M|^2 \times BF$ in order to have more counts π is better because $|M_\pi|^2 \propto |V_{ud}|^2 \gg |M_D|^2 \propto |V_{us}|^2$

To see that they had Kaons and not pions they used a Cherenkov detector (made of aerogel) and by identifying the final state they were able to say wheter they saw a B^0 or a $\bar{B}^0$. They measured the number of events looking for a difference for the 2 process (indication for a CP violation). It took a while and in 2004 they found:

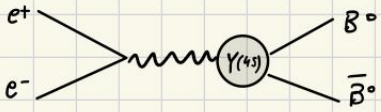


$$n\,(B^0 \longrightarrow K^+\pi^-) = 910$$
$$n\,(\bar{B}^0 \longrightarrow K^-\pi^+) = 696$$

$$A_{symm} = \frac{\Gamma(\bar{B}^0 \to K^-\pi^+) - \Gamma(B^0 \to K^+\pi^-)}{\Gamma(\bar{B}^0 \to K^-\pi^+) + \Gamma(B^0 \to K^+\pi^-)} \simeq -0.133 \pm 0.03 \pm 0.009$$

Direct proof of CP violation!

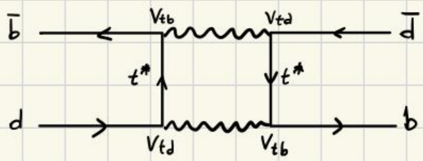
One interesting thing here is the quantum entanglement. The process we have is:



The B^0 and $\bar{B}^0$ have $S=0$ (because they are scalar bosons). Then since the initial state ha $J=1$ this means that in order to conserve J, in the final state of B mesons we should have $L(B^0\bar{B}^0)=1$. The wavefunction of the final state is:

$$\psi(B^0\bar{B}^0) = \psi_{space}\,\psi_{flavor} = (-1)^L\,\psi_{flavor} = -\psi_{flavor}$$

Since they are 2 bosons, it must be symmetric. Therefore ψ_{flavor} must be antisymmetric. In a collision we produce 2 physical particles B^0 and $\bar{B}^0$. These are free particles and we have no control over them. We know that due to Weak interactions these 2 particles can oscillate one into the other



If it happens that a B^0 oscillates and becomes a $\bar{B}^0$, since the flavor wavefunction must be antisymmetric then immediately $\bar{B}^0$ oscillates into B^0. There is no exchange of information between the 2 particles, they are travelling in space and time separately but if one changes flavor then the other has to do it immediately. This is an example of the Einstein - Podolsky - Rosen paradox : we produce 2 independent physical states but because of quantum entanglement they have to obey each other rules, if one flip flavor then the other has to do it immediately.
Moreover they can decay with a lifetime of $\tau_B \simeq 1.54\,ps$. The decay of one of the two particles breaks the entanglement and the other one is free to do what it wants (there's no entanglement if there is no flavor).

$B^\circ - \bar{B}^\circ$ MIXING

As we already seen in the case of the K° the time evolution of $B^\circ/\bar{B}^\circ$ is governed by a complex matrix that can be written in terms of hermitian matrices M and Γ:

$$H = \begin{pmatrix} M - i\frac{\Gamma}{2} & M_{12} - i\frac{\Gamma_{12}}{2} \\ M_{12}^* - i\frac{\Gamma_{12}^*}{2} & M - i\frac{\Gamma}{2} \end{pmatrix}$$

Its eigenstates can be written in terms of the flavor eigenstates $B^\circ/\bar{B}^\circ$ (the ones produced by nature. In general they are

$$\begin{cases} |B_L\rangle = p|B^\circ\rangle + q|\bar{B}^\circ\rangle \\ |B_H\rangle = p|B^\circ\rangle - q|\bar{B}^\circ\rangle \end{cases} \qquad \text{with} \quad p^2+q^2=1$$

The most conspicous feature of $K^\circ - \bar{K}^\circ$ mixing is the disparate lifetimes of the two mass eigenstates and this is why we called them K_L and K_S. In this case instead the lifetime difference is quite small (this allows us to neglect Γ_{12} (for B_d but not for B_s) while the mass difference of the two mass eigenstate is not negligible, $\Delta m = 0.32$ MeV (this is why we called the 2 eigenstates B_L (light) and B_H (heavy).

$$\mu_L = M - i\frac{\Gamma}{2} - |M_{12}|$$

$$\mu_H = M - i\frac{\Gamma}{2} + |M_{12}|$$

so that the mass splitting is $\Delta m = 2|M_{12}|$. The eigenstates are

$$\begin{cases} |B_L\rangle = \frac{1}{\sqrt{2}}\left(|B^\circ\rangle - \frac{|M_{12}|}{M_{12}}|\bar{B}^\circ\rangle\right) \\ |B_H\rangle = \frac{1}{\sqrt{2}}\left(|B^\circ\rangle + \frac{|M_{12}|}{M_{12}}|\bar{B}^\circ\rangle\right) \end{cases}$$

and they evolve simply as:

$$\begin{cases} |B_L(t)\rangle = e^{-i\left(M - \frac{\Delta m}{2} - i\frac{\Gamma}{2}\right)t}|B_L\rangle \\ |B_H(t)\rangle = e^{-i\left(M + \frac{\Delta m}{2} - i\frac{\Gamma}{2}\right)t}|B_H\rangle \end{cases}$$

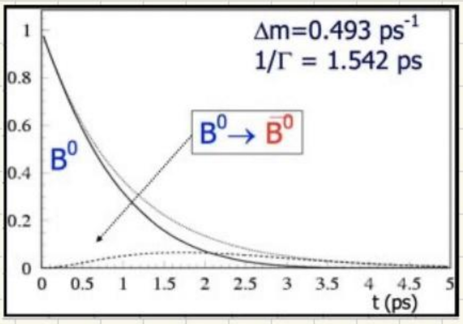
So a state that at $t=0$ is purely B° (and $\bar{B}^\circ$) will oscillates as:

$$\begin{cases} |B^\circ(t)\rangle = e^{-i\left(M - i\frac{\Gamma}{2}\right)t}\left[\cos\left(\frac{\Delta m}{2}t\right)|B^\circ\rangle - i\frac{|M_{12}|}{M_{12}}\left(\sin\frac{\Delta m}{2}t\right)|\bar{B}^\circ\rangle\right] \\ |\bar{B}^\circ(t)\rangle = e^{-i\left(M - i\frac{\Gamma}{2}\right)t}\left[\cos\left(\frac{\Delta m}{2}t\right)|\bar{B}^\circ\rangle - i\frac{|M_{12}|}{M_{12}}\left(\sin\frac{\Delta m}{2}t\right)|B^\circ\rangle\right] \end{cases}$$

Therefore:

$$\boxed{\begin{aligned} P(B^\circ \text{ remains } B^\circ, t) &= \frac{e^{-\Gamma t}}{2}\left(1 + \cos(\Delta m t)\right) \\ P(B^\circ \text{ oscillates into } \bar{B}^\circ, t) &= \frac{e^{-\Gamma t}}{2}\left(1 - \cos(\Delta m t)\right) \end{aligned}}$$

The function above is shown in the plot:



To measure this behaviour one defines what is called the mixing asymmetry:

$$\boxed{\text{Mixing asymmetry} = \frac{\# B^0 - \# \bar{B}^0}{\# B^0 + \# \bar{B}^0}}$$

The fact is that with an e^+e^- collision we produce a pair of B mesons:

$$e^+e^- \longrightarrow B^0\bar{B}^0 \qquad t=0$$

then at $t>0$ we can have:

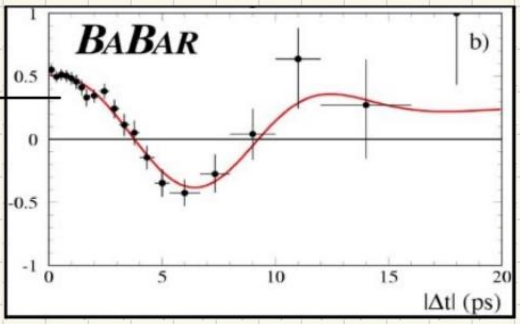
- $B^0\bar{B}^0$ (no oscillation)
- B^0B^0 ($\bar{B}^0 \longrightarrow B^0$ oscillation)
- $\bar{B}^0\bar{B}^0$ ($B^0 \longrightarrow \bar{B}^0$ oscillation)

With t_1, t_2 decay times of the two B mesons, and with $\Delta t = t_2 - t_1$ we can define the amplitude of mixing (oscillation) as:

$$\boxed{A \equiv \frac{(\# B^0B^0 + \# B^0\bar{B}^0) - \# B^0\bar{B}^0}{(\# B^0B^0 + \# B^0\bar{B}^0) + \# B^0\bar{B}^0} = \cos(\Delta m \cdot \Delta t)}$$

and this is what they measured:

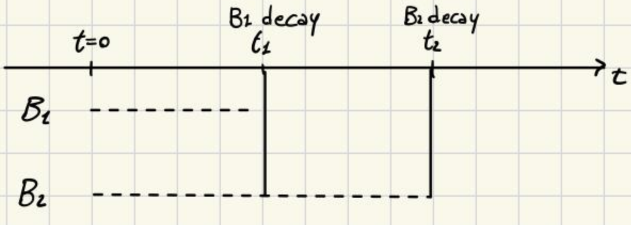
a lot of events at small times (because here the particles have not decayed yet.



So thank to quantom entanglement we can measure the $B^0\bar{B}^0$ oscillations in time. The difficolty is that we don't know which B^0 decays first: it is a random process. However experimentally we can understand who decayed first by looking at the decay products:

$$B^0 \longrightarrow D^-K^+,\ D^-\pi^+ \qquad \bar{B}^0 \longrightarrow D^+K^-,\ D^+\pi^-$$

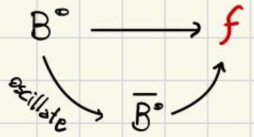
We start with two B mesons B_1 and B_2 and we don't know which one of the two is B^0 and which $\bar{B}^0$



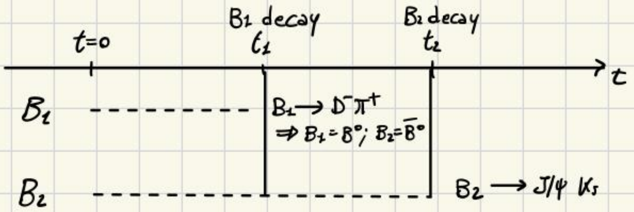
When the 1^{st} particle decay looking at its decay products we can understand whether it was a B° or a $\overline{B}^\circ$. Then after the 1^{st} decay the 2^{nd} one is no more entangled, it is free to either oscillate or not. Then at the time t_2 we can look again at the decay products and we can either find that the particle has oscillated or not. That is how we count the number of oscillations as a function of the time difference $\Delta t = t_2 - t_1$.

INTERFERENCE BETWEEN DECAYS AND MIXING

In this case we have that to achieve f we have 2 possible paths:



In optics the interference is parametrized by the distance between the 2 holes; here we have the CKM matrix element which produce the phase difference between the 2 diagrams (the one starting from B° and the one starting from $\overline{B}^\circ$). From the collision we get 2 B mesons B_1 and B_2 but we don't know which one is B° and $\overline{B}^\circ$. As before we look at the decay products



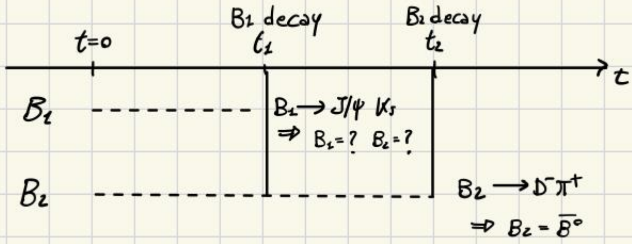
At this point 2 things could have happened:

1) B_2 was a $\overline{B}^\circ$, it remained a $\overline{B}^\circ$ and then it decayed to $J/\psi\, K_S$
2) B_2 was a $\overline{B}^\circ$, it oscillated to a B° and it decayed to $J/\psi\, K_S$

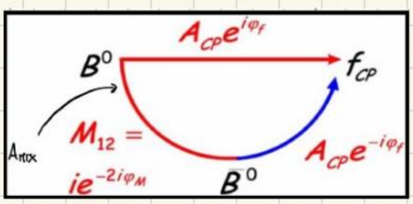
} we have no idea of which one of the 2 has happened and we have to take into account

Here B_1 is called B_{tag} that is we tag/identify the flavor of B_1 and of B_2 at t_1.

The other thing that could happen is the following



This means that when we observe B_2 decay into a final state to obtain the amplitude for such a transition we have to sum the amplitude of the 2 different processes that can happen (with or without oscillation):



$A_{CP}: B^\circ \longrightarrow f_{CP} = |A_{CP}|e^{i\varphi_f}$

$\overline{A}_{CP}: \overline{B}^\circ \longrightarrow f_{CP} = |A_{CP}|e^{-i\varphi_f}$

$A_{MIX}: B^\circ \longleftrightarrow \overline{B}^\circ = |A_{MIX}|e^{i\varphi_m}$

N.B. The reason behind the choice of $f_{CP} = J/\psi\, K_S$ is because we can clearly see it in the detector.

$J/\psi \ K_s$

$K_s \longrightarrow \pi^+\pi^-$

$J/\psi \longrightarrow \ell^+\ell^-, e^+e^-, \mu^+\mu^-$

$\longrightarrow$ we observe 4 tracks of charged particles (2 positive and 2 negative) with the right invariant mass. and the constraints $m_{ee} = m_{J/\psi}$, $m_{\pi\pi} = m_{K_s}$

We like this event also from a theoretical point of view. Looking at the amplitude for the 1st process $A_1 = A_{CP}$ we have to look at the following diagram

Looking at the CKM matrix we can see that this diagram has no complex phase inside and that is good.

Then we can look at the amplitude A_2 for the second process that can happen $B^0 \longrightarrow \bar{B}^0 \longrightarrow J/\psi K_s$

Here there is a K^0 but we talk about K_s because experimentally it is very easy to distinguish between K_s and K_L

An important parameter to understand the probability of oscillation + decay is $\lambda = \frac{q}{p}\frac{\bar{A}}{A}$ where q and p are the ones of the combination:

$$B_L = p|B^0\rangle + q|\bar{B}^0\rangle \quad ; \quad B_H = p|B^0\rangle - q|B^0\rangle \qquad q^2+p^2=1$$

What we found experimentally is that:

- $|q|/|p| \simeq 1$ at 10^{-4} precision, so there is no CP violation in the mixing
- $|\bar{A}|/|A| \simeq 1$ at 1% precision

So if λ is there it has to be a complex phase : $\lambda = e^{-i2\beta}$ where β is the phase in the CKM matrix. This fact is important and if we combine the time evolution of the oscillation and of the decay what we find is that the probability of having a B^0 or a $\bar{B}^0$ after a time Δt is :

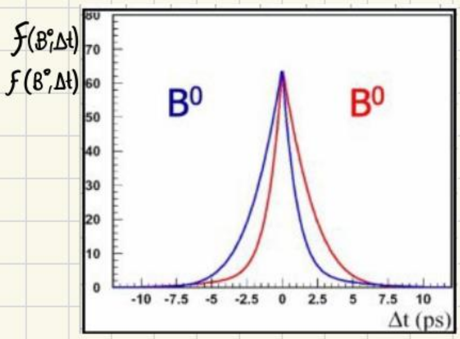
$$\boxed{\begin{aligned} f(B^0, \Delta t) &= \frac{e^{-t/\tau}}{2}\left(1 + Im\,\lambda \cdot \sin(\Delta m\,\Delta t)\right) \\ f(\bar{B}^0, \Delta t) &= \frac{e^{-t/\tau}}{2}\left(1 - Im\,\lambda \cdot \sin(\Delta m\,\Delta t)\right) \end{aligned} \qquad Im\,\lambda = \sin 2\beta}$$

Then what we can define is the CP Asymmetry as:

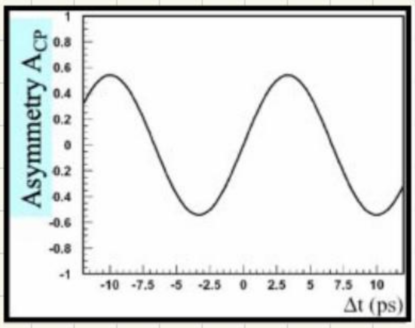
$$\boxed{\text{CP Asymmetry} = \frac{|\bar{A}|^2 - |A|^2}{|\bar{A}|^2 + |A|^2} = \frac{\#(B^0 \to J/\psi K_s) - \#(\bar{B}^0 \to J/\psi K_s)}{\#(B^0 \to J/\psi K_s) + \#(\bar{B}^0 \to J/\psi K_s)} = \sin(2\beta)\sin(\Delta m \cdot \Delta t)}$$

That is easy to measure experimentally, we just have to count B^0 and $\bar{B}^0$ as a function of $\Delta t = t_{cp} - t_{tag}$. So what we have to measure is the amplitude of these oscillations so that we can have an estimate of β. If the oscillation amplitude is $\neq 0$ then also β, hence a complex phase exists. If instead the amplitude is compatible with 0 then also β is and the CKM mechanism is broken.

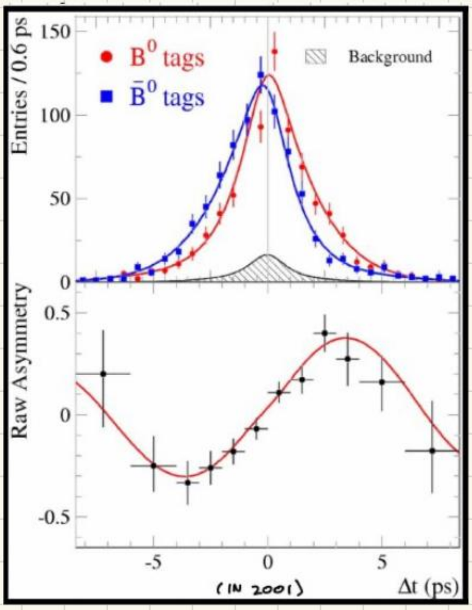
The functions that we have to measure are:



from these we should see the oscillations on the right $\longrightarrow$



So what they did was counting J/ψ and K_S as a function of time and what they saw in 2001 was:



$\sin 2\beta \neq 0$ $\longrightarrow$ $\boxed{\sin 2\beta = 0.741 \pm 0.067_{stat} \pm 0.033_{syst}}$

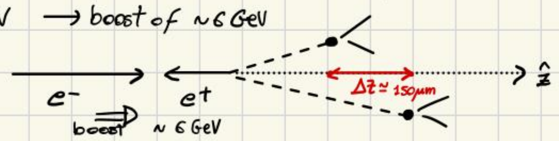
This proves that the KM mechanism works and for this discovery Kobayashi and Maskawa got the Nobel prize in 2008. One can also do the experiment with $J/\psi\, K_L$ with the difference that the curve is opposite since what changes is the CP sign.

We proved all the prediction of the KM mechanism to be true but there is still a problem: the amount of observed CP violation is not sufficient to explain the current asymmetry we see in nature, we're not able to explain still the asymmetry between matter and antimatter in the universe.

This entanglement gives us a way to see that the CKM matrix element is complex. What they did at Slac was putting the machine at the mass of $Y(4S)$ $\rightarrow \sqrt{s} = 10.58$ GeV. looking for the process:

$$e^+e^- \longrightarrow Y(4S) \longrightarrow B^0\bar{B}^0 \longrightarrow Q = \sqrt{s} - 2m_B = 300 \text{ MeV}$$

So there is a little energy left for the 2 particles in the final state, hence the boost is small: $\beta\gamma = \frac{P}{m} \simeq \frac{300\text{MeV}}{5.28\text{GeV}}$
So the B mesons are practically at rest and since we have $S=0$, the decay is isotropic.
This fact was used to throw away background events.
However the fact that the 2 mesons are practically at rest is a problem because they overlap and we instead want to measure the distance between the 2 particles to use the quantum entanglement of the B^0 and $\bar{B}^0$. To solve this problem what they did was using asymmetric beams with energies $E_{e^-} \simeq 9$ GeV, $E_{e^+} = 3.1$ GeV $\longrightarrow$ boost of ~ 6 GeV

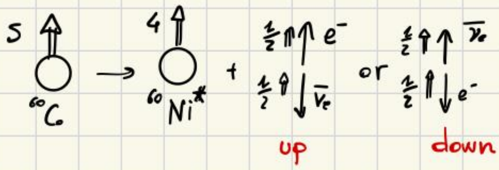


Now the longitudinal distance between the two decay points of B^0 and $\bar{B}^0$ became experimentally measurable : $\langle \Delta z \rangle \simeq 150 \mu m$ (and therefore a measurable decay time)

FRANCESCO PANDOLFI's part

Goldhaber experiment (1957)

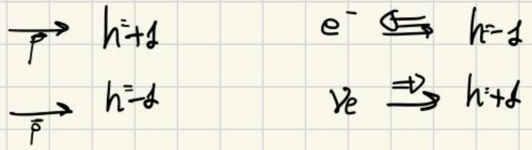
In 1956 there was WU exp $\longrightarrow$ Weak cross section can be a function of $\vec{\sigma}\cdot\vec{p}$



In up we have that the e^- has the spin in the same dir of $\vec{p}$ and $\bar{\nu}_e$ in the opposite dir.
In down we have the opposite situation
Why these 2 conf should be diff.? We find that down is favored.
If I define a quantity called helicity:

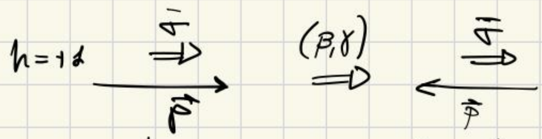
$$h = \frac{\vec{\sigma}\cdot\vec{p}}{|\vec{\sigma}|\cdot|\vec{p}|}$$

is basically telling us the angle between $\vec{\sigma}$ and $\vec{p}$.



w. int $\longrightarrow$ violates parity

$\vec{p}$ is not a L. inv I could flip it with a boost:



$\longrightarrow$ h is not invariant

If I have 0 mass h is invariant because $v=c$ and I can't flip $\vec{p}$.
In the S.M. $m_\nu = 0$

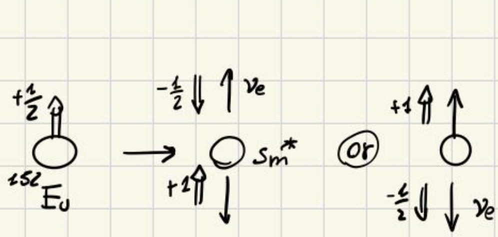
W. int. can distinguish states that depends $\vec{\sigma}\cdot\vec{p}$. The aim of Goldhaber is show if there's a preferred helicity for ν $h=+1$ or $h=-1$

ν: difficult to measure, Smart idea:

The exp. is based on ^{152}Eu. There is an e^- capture and a production of ν_e and $^{152}Sm^*$.

$$^{152}Eu + e^- \longrightarrow \nu_e + {}^{152}Sm^* \qquad (\text{similar } p + e^- \longrightarrow \nu_e + n)$$

Europium has spin 1/2



In the 1st conf. $h_{\nu_e} = -1$ and $h_{Sm} = -1$

In the 2nd conf $h_{\nu_e} = +1$ and $h_{Sm} = +1$

Trick

$\Longrightarrow \boxed{h(\nu_e) = h(Sm)}$!

Sm is in an excited states

N.B.		
STR.	$\rightarrow$	$\tau \sim 10^{-22}s / 10^{-23}s$
EM	$\rightarrow$	$\tau \sim 10^{-12} / 10^{-15}s$
WEAK	$\rightarrow$	$\tau \sim 10^{-6} / 10^{-8}s$

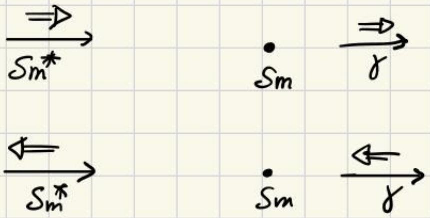
$$Sm^* \xrightarrow{E.M \rightarrow \tau \sim 10^{-4}s \; 152} Sm + \gamma$$

$s=1$ $\quad$ $s=0$ $\quad$ $s=1$ $\rightarrow$ the spin is transfered to γ

If γ is emitted in the same direction of Sm^* :

case $h=+1$

case $h=-1$



We know $h(\nu) = h(Sm^*) = h(\gamma)$ $\rightarrow$ we transfer the h measurement from ν_e to Sm to γ.
From an energy point of view

$$Sm^* \longrightarrow Sm + \gamma$$

Sm^*
Sm $\quad$ $\Delta E = 961$ KeV $\quad$ width $\sim 10^{-2}$ eV

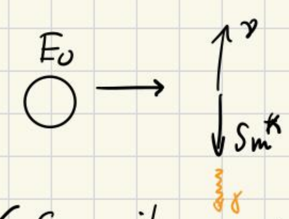
If the γ reuncounter a Sm it cannot reexites it

$$\gamma + Sm \longrightarrow Sm^*$$

γ is a little bit of less energetic of 3,2 eV because of the recoil of Sm.

In the ref. frame of Sm we have $Sm^* \rightarrow \gamma, Sm$ $\rightarrow E(\gamma) = \Delta E - 3.2eV < \Delta E\,(Sm, Sm^*)$

However Sm^* is produced by the Eu decay. It turns out that the boost of Sm^* is enough to compensate for the 3.2 eV loss due to recoil if the γ is emitted in the same direction of Sm^*



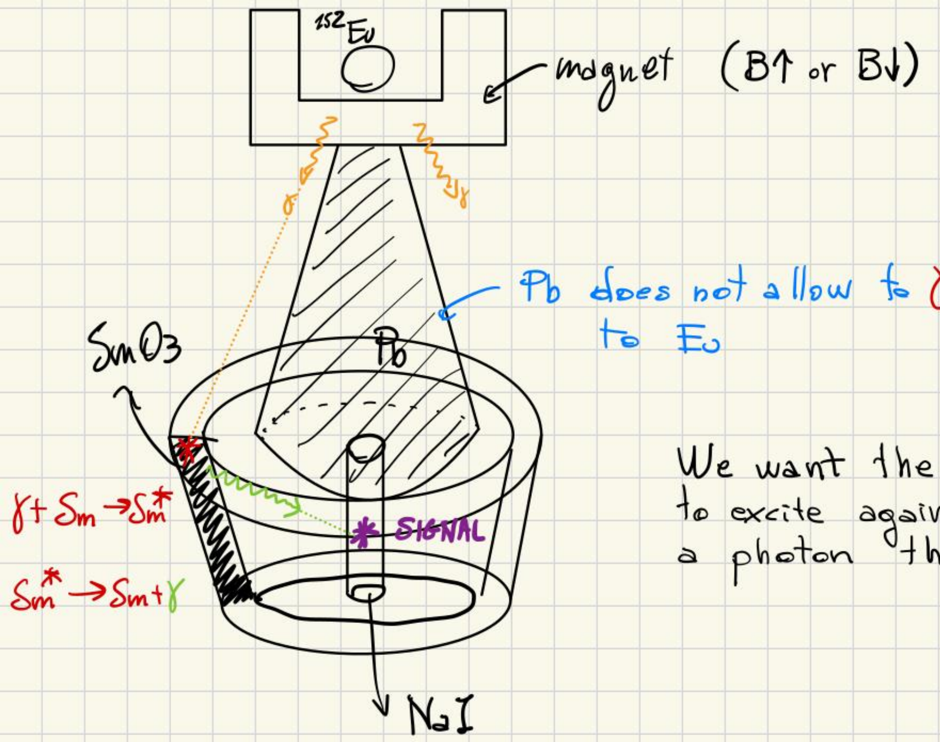
If γ is emitted in the same dir of Sm it regains some en. thanks to L. boost so it can reexites Sm

<u>Recap:</u>

<u>tricks 1</u> $\quad$ <u>tricks 2</u>

If $\gamma \parallel \vec{p}(Sm^*)$ $\Rightarrow$ 1) $h(\gamma) = h(\nu_e)$ and 2) $E = \Delta E\,(Sm, Sm^*)$

APPARATUS



We want the photon emitted by the Sm^* to excite again another Sm that then will emit a photon that we can count

CASES:

If $\boxed{h_\nu = +1}$

$B\uparrow$: $+\frac{1}{2}\Uparrow$ Eu $\mapsto$ $+1 \Uparrow \uparrow Sm^*$, $-\frac{1}{2}\Downarrow \downarrow \nu$ $\mapsto$ $\uparrow Sm$, $+1 \Uparrow \downarrow \gamma$ $h=-1$

→ no boost
→ $E_\gamma < \Delta E(Sm, Sm^*)$
No signal

$B\downarrow$: $-\frac{1}{2}\Downarrow$ Eu $\mapsto$ $+\frac{1}{2}\Uparrow \uparrow \nu$, $-1 \Downarrow \downarrow Sm^*$ $\mapsto$ $\uparrow Sm$, $-1 \Downarrow \downarrow \gamma$ $h=+1$

→ yes bost
→ $E_\gamma > \Delta E$
yes signal

If $\boxed{h_\nu = -1}$

$B\uparrow$: $+\frac{1}{2}\Uparrow$ Eu $\to$ $-\frac{1}{2}\Downarrow \uparrow \nu_e$, $+1 \Uparrow \downarrow Sm^*$ $\mapsto$ $\uparrow Sm$, $\Uparrow \downarrow \gamma$ $h=-1$

Yes Boost
→ Yes signal

$B\downarrow$: $\Downarrow$ Eu $\mapsto$ $\Downarrow \uparrow Sm^*$, $\Uparrow \downarrow \nu_e$ $\mapsto$ $\uparrow Sm$, $\Downarrow \downarrow \gamma$ $h=+1$

no Boost
→ no signal

	$h_\nu = +1$	$h_\nu = -1$
$B\uparrow$	no counts	Yes counts
$B\downarrow$	~~Yes counts~~	no counts

$\Longrightarrow$ $h_\nu = -1$

It is not seen ⟶ found asymmetry

So we only see $\nu_e \xrightarrow[\vec{p}]{\overset{\vec{s}}{\Leftarrow}} h_\nu = -1$ ~~$\xrightarrow[\vec{p}]{\overset{\vec{s}}{\Rightarrow}} h_\nu = +1$~~

$q = 0$ ~~EM~~
$c = 0$ ~~str~~ $\longrightarrow$ ν with $h_\nu = +1$ does not exist
$m = 0$ ~~grav~~
$h = -1$ weak

If we try to extend this theory of w. int. to other particles we have a problem for a massive particle h is not invariant indeed it depends on the frame. We need to find another physics obs. capable to prove the same thing which is not dependent by the frame: chirality γ^5

We can split the wavefunction into a left and right part as:

$$\psi(x) = \underbrace{\tfrac{1}{2}(1+\gamma^5)}_{P_R}\psi(x) + \underbrace{\tfrac{1}{2}(1-\gamma^5)}_{P_L}\psi(x) = \psi_R(x) + \psi_L(x)$$

It turns out that for massless particles $\psi_R(x)$ has $h = +1$ and $\psi_L(x)$ has $h=-1$

So what we could say is that weak int. only interacts with the left part of the wave function and this is exactly the case. It turns to be true also for massive particles.

In the massive case instead $h \neq$ chirality and we have to take into account both chirality.
For a massive particle if $h = -1$ then the left part of the wave func. is weighted by $(1+\beta)$ while the right part by $(1-\beta)$. And viceversa for $h = +1$ massive particles
This implies that in the massless limit $\beta \sim 1$ w. int interact only with $h=-1$ particles

Let's now take for example the π decay:

$\pi^- \longrightarrow \mu^- \bar{\nu}_\mu$ $\qquad$ $m(\pi) \sim 140$ MeV
$\qquad\qquad\qquad\qquad$ $m(\mu) \sim 106$ MeV
$\pi^- \longrightarrow e^- \bar{\nu}_e$ $\qquad$ $m(e) \sim 0.5$ MeV

the e^- channel should be favored due to phase space but it turns out the opposite situation. To understand why let's look at the situation in the π rest frame

$+\frac{1}{2} \Uparrow \uparrow \bar{\nu}_e$ $\quad h_\nu = +1$ $\longrightarrow$ $\bar{\nu}_e$ (since $m=0$) has $h=+1$

$\pi^- \bullet \longrightarrow$
$s=0$

$+\frac{1}{2} \Downarrow \downarrow e^-$ $\quad h_e = +1$ $\longrightarrow$ since the decay is back to back and $s_\pi = 0$ then e^- is forced to have $h = +1$

Since e^- is not massless it has left component even if $h=+1$. However since by computation $\beta_e \sim 0.99997$ the left part is suppressed by a factor $(1-\beta)$

But in w. int for particles only the ψ_L participates to the game and therefore this decay is heavily suppressed ($\langle i | M | f \rangle$ very small)

For the μ instead $\beta_\mu \sim 0.27$ and the muonic decay is not suppressed

What one finds experimentally is that

$$R_\pi = \frac{\pi \longrightarrow e\nu}{\pi \longrightarrow \mu\nu} \sim 10^{-4}$$

N.B. Weak int. only interacts with left component of the particle (right for antiparticle).

PARTICLES

$h = -1$

$\Leftarrow$

$\longrightarrow$

$\psi_L \sim \frac{1}{2}(1+\beta)$

$\psi_R \sim \frac{1}{2}(1-\beta)$

ANTIPARTICLES

$h = +1$

$\Rightarrow$

$\longrightarrow$

$\psi_L \sim \frac{1}{2}(1-\beta)$

$\psi_R \sim \frac{1}{2}(1+\beta)$

Appendice

$\longrightarrow \quad \frac{d\sigma}{d\Omega} = \underbrace{(\dots)\frac{\alpha^2}{E^4 \sin^4\frac{\theta}{2}}}_{\text{Mott}} \frac{E'}{E} \underbrace{E'^2\left(1-\beta^2 \sin^2\frac{\theta}{2}\right)}_{\text{relativistic + spin}} \quad ; \quad \frac{d\sigma}{dE'} = \frac{\#\text{events}}{\Delta E'}$

$$\longrightarrow \quad \boxed{\sigma = 2\pi \int d\theta \sin\theta \quad \frac{1}{E^4} \frac{1}{\sin^4\frac{\theta}{2}} \frac{1}{1+2\frac{E}{M}\sin^2\frac{\theta}{2}} E'^2\left(1-\beta^2\sin^2\frac{\theta}{2}\right)}$$

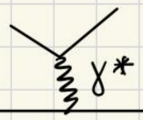
In practice:

$$\sigma = \int_{\theta_0-\varepsilon}^{\theta_0+\varepsilon} \frac{d\sigma}{d\Omega}\, d\Omega \qquad \varepsilon = 3\sigma \text{ resolution}$$

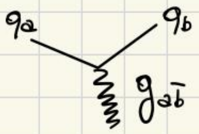
Mass of the quarks

Mass of a particle: energy of the particle in the rest frame.
Since there is not single color state we cannot have a single q. state → we cannot define the mass of a single quark

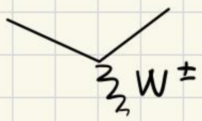
Q.C.D. allows us to compute the binding energy in a composite object such as Baryons and Mesons. This binding force is related to the color charge.

QED 

$\alpha_{QED} = \frac{1}{137}$ 1 photon massless $Q=0$

QCD 

$\alpha_{QCD} \simeq 0{,}1$ 8 gluons massless $Q=0$ colored

WEAK 

$\alpha_{weak} = ?$ 2 charged vector bosons $\begin{cases} W^\pm & m \simeq 80 \text{ GeV} \\ Z^0 & m \simeq 90 \text{ GeV} \end{cases}$